Econometrics of Constructing Panels of Real Incomes and Inequality
and
Economic Welfare and Convergence Over the Last Fifty Years

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(jointly with Alicia Rambaldi, Reza Hajargasht and Bill Griffiths)

Sen's (1976) framework for welfare comparisons

A rephrased version of theorem T.15 in the paper is:

Let ***x*** and ***x'*** are two vectors enjoyed by two communities not necessarily of the same size but with the same types of commodities. Let μ and μ' are mean (per capita) incomes adjusted for price level differences and G and G' the respective Gini coefficients of the two income distributions of the two communities.

Under a set of axioms on preferences, $\mu(1-G) > \mu'(1-G')$ implies that ***x*** *P* ***x'***

To make welfare comparisons, we need:

- For temporal comparisons, need mean incomes at constant prices – incomes adjusted using CPI or another deflator
- For spatial comparisons across countries – need *purchasing power parities of currencies or spatial price index numbers and real per capita incomes in PPP terms*
- Gini measure of inequality

Purchasing Power Parities (PPPs), Real Per Capita Incomes and Inequality

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- PPPs are amounts of currencies, of different countries, that have the same purchasing power as one unit of a reference currency (e.g. US$) with respect to a selected basket of goods and services.
- Real per capita income is per capita income in national currency units converted into dollars using PPPs.
- Income distributions – lognormal; Generalised beta-2; etc
- Inequality measures – Gini coefficient and Theil's Inequality Measure

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PPPs, Real Per Capita Incomes and Inequality (selected countries, 2017)

Country	XR	PPP_GDP	PLI = PPP/XR*100	per capita real GDP	Gini
Australia	1.31	1.47	112.34	50,153	0.3444
China	6.76	4.18	61.90	14,150	0.3884
Hong Kong	7.79	6.01	77.13	59,927	0.539
India	65.12	6.01	9.23	6,149	0.3621
Japan	112.66	105.38	93.54	40,827	0.3316
USA	1.00	1.00	100.00	59,984	0.4248

World	
Real per capita GDP	16359
Gini coefficient	0.5887
International Poverty Line	$2.15
Number of Global Poor	648.10 mill

Main Sources of PPPs and Inequality Measures

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- PPPs from the International Comparison Program at the World Bank
  - Compiled periodically, roughly once in 5 years
  - The last round was for 2017 and the 2021 round being completed.
  - Variable coverage and incomplete panel of PPPs
- Penn World Tables – a complete panel of PPPs
  - First set of extrapolations of PPPs for 100 countries for the year 1970 was produced by Kravis, Heston and Summers (1978)
  - Available since 1980's; the latest version PWT 10.0 released in 2021
  - Covers more than150 countries and a 70-year period
  - Extrapolations of benchmark PPPs
    - Inter and extrapolation making limited use of benchmark PPPs from ICP
    - Uses movements in national price levels
  - PWT widely used and most cited (Summers and Heston,1991)

Main Sources of PPPs and Inequality Measures

Inequality databases

- World Bank POVCAL data
- World Inequality Database (WID)
- Income Distribution Database, OECD
- Standardized World Income Inequality Database (SWIID)
- World Income Inequality Database (WIID)

Incomplete panel of income distribution data

University of Queensland International Comparison Database (UQICD) Version 3.0

A new series (panels) on PPPs, real incomes, inequality, regional growth and inflation

URL: uqicd.economics.uq.edu.au

UQICD V3.0

UQ International Comparisons Database

Real Incomes by Country

Real Incomes by Region

Inequality by Country

Inequality by Region

Timeline for UQICD Development

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- The idea of compiling an alternative to PWT using state-space approach dates back to 1997!
- Work on state-space approach started around 2005. (Doran, Rambaldi and Rao; Rambaldi has been driving this work since 2009)
- UQICD Version 1.0 was released in 2011 – limited to just PPPs and real incomes
- Work on fitting income distributions with limited data started in 1995, gained momentum since 2004. (Brice, Chotikapanich, Griffiths, Hajargasht and Rao
- Work on panels of income distributions started in 2014. Hajargasht fitted and compiled all the income distributions available in UQICD)
- Work on measuring global growth and inflation started in 2015 by Rao and Balk, joined by Rambaldi (Balk, Rambaldi and Rao, 2022)
- These developments have contributed to the development of the three modules of UQICD Version 3.0

Econometric Methodology for the Panel of PPPs and Real Incomes

ICP Benchmarks – country participation

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ICP Phase	Benchmark year	No. of participating countries
I	1970	10
II	1973	16
III	1975	34
IV	1980	60
V	1985	64
VI	1993	117
VII	2005	146
VIII	2011	177
IX	2017	176

Incomplete panel of PPPs; Sparse in early years; In addition, OECD and Eurostat compile PPPs for their member countries every three years

Combining Theory and Noisy Data

Sources of information for $p_{it} = ln(PPP_{it})$

1. **ICP Benchmark PPPs**: Observation of the variable of interest contaminated with noise
2. **A Model Derived from the Theory of Price Levels**: Links national level data to variable of interest.
3. **Price movements from national sources in the form of deflators**: Links national accounts data to variable of interest
4. **Reference Country Definition**: A restriction that must hold, $p_{reference\ country,t} = 0$

Our Approach

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- **Use all available benchmark information – an unbalanced panel**
- **Set up an econometric model to predict PPP_i *combining ICP benchmark with other available information***
- **Write it in a state-space form**
- **Use a Spatio-Temporal filter to produce predictions and associated standard errors**

Combining Theory and Noisy Data (Source 1)

Surveys are very resource intensive,

Carried out by national statistical agency of those countries that participate in the ICP.

Internationally comparable basket is priced

We can then write

$$\tilde{p}_{it} = p_{it} + \xi_{1it}$$

where,

$\tilde{p}_{it}$ ICP benchmark observation for participating country i at time t

ξ_{1it} is a random error accounting for measurement error.

The Theory of Price Levels

National Price Level differences (or exchange rate deviation index – PPP/Xr) are due to:

productivity differences in traded and non-traded goods sectors across developed and developing countries.

Some of the primary drivers of Price Levels:

Size of the agriculture sector in the economy, openness, educational attainment, share of exportable services (such as tourism), resource abundance, size of the population, trade balance.

Combining Theory and Noisy Data (Source 2)

$$r_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_{it} + u_{it}$$

where,

$r_{it} = \ln(PPP_{it} / ER_{it})$; $\mathbf{x}'_{it}$ a set of conditioning variables

$\boldsymbol{\beta}_{it}$ a vector of parameters

u_{it} a random disturbance with specific distributional characteristics

We obtain a prediction:

$$\hat{p}_{it} = \mathbf{x}'_{it}\hat{\boldsymbol{\beta}}_{it} + \ln(ER_{it})$$

Combining Theory and Noisy Data (Source 3)

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We assume some measurement error exists in national accounts and thus use

$$PPP_{i,t} = PPP_{i,t-1} \times \frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}}$$

to define:

$$p_{it} = p_{i,t\text{-}1} + c_{it} + \eta_{it}$$

where,

$$\mathrm{c}_{it} = \ln\left(\frac{GDPDef_{i,[t,t-1]}}{GDPDef_{US,[t,t-1]}}\right)$$

η_{it} is a random error accounting for measurement error in the growth rates

A Complete Panel of PPPs (and Real Incomes)

The UQICD Econometric Approach: Combining Information from a number of "noisy" sources

1. A constraint, the PPP of the reference country is equal to 1 at every time period

$$P_{USA,t} = 1; \ \ t = 1,2,\dots,T$$

2. Observed ICP PPPs Incomplete panel, $\tilde{P}_{jt}$:

- participating countries,
- benchmark years

Assumed to be measured with error

3. Predictions ($\hat{P}_{jt}$) from an economic model of price levels,

$$ln\left(PPP_{jt}/XR_{jt}\right) = \beta_{0t} + \boldsymbol{x}'_{jt}\beta_s + e_{jt}$$

4. PPPs temporal evolution

$$PPP_{jt} = PPP_{j,t-1} \times \left(\frac{GDPDef_{j,|t-1,t|}}{GDPDef_{USA,|t-1,t|}}\right)$$

Assumed to be measured with error

Stacking "observed/measured" (log transformed)

$$\begin{bmatrix} \mathbf{0} \\ \tilde{\boldsymbol{p}}_{jt} \\ \hat{\boldsymbol{p}}_{jt} \end{bmatrix} = \boldsymbol{y}_t$$

1. The vector has *N* rows == number of countries including the reference (USA)
2. Measurement in a state-space system

$$\boldsymbol{y}_t = Z_t\boldsymbol{p}_t + B_t X_t \theta + \boldsymbol{\zeta}_t$$

p_t is the unobserved log PPP (state vector)

3. Temporal movement, country *j*

$$p_{jt} = p_{j,t-1} + \left\{ ln\left(\frac{GDPDef_{j,|t-1,t|}}{GDPDef_{USA,|t-1,t|}}\right) + \eta_{jt} \right\}$$

Outcome: A panel of estimated $\widehat{PPP}_{jt}$ and $\mathrm{SE}(\widehat{PPP}_{jt})$ for N=185 countries for 1970-2019

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Analytical Properties of the UQICD Approach

1. **The Estimated PPPs are invariant to the choice of reference country**
2. **The estimated PPPs are weighted sums of the information in y_t**
3. **If regression predictions are switched off, the predicted PPPs are "weighted averages" of benchmark-year only extrapolations.**
4. **Special Cases**
   - Constraining the model to track PPPs for countries participating in the ICP benchmarks
   - Constraining the model to preserve movements in the Implicit GDP Deflator

Results and proofs provided in various papers, please consult the UQICD User Guide

Series available in UQICD Module 1

1. **PPPs and Real Incomes (RGDP - current prices)**
   - GDP Level
   - Household Consumption (CONS)
   - Government Expenditures (GOV)
   - Gross Capital Formation (GCF)
2. **CRGDP (Real Incomes in 2011 constant) – See Appendix C of UQICD User Guide for details**

Construction of Panel of Income Distributions

Main Objective

- A panel of yearly income distributions is provided for 159 countries for years 1970-2019.
- The distributions are estimated from aggregate grouped data using four alternative functional forms.
    - Lognormal distribution
    - Pareto-lognormal distribution
    - Generalised beta distribution of the second kind
    - Mixture of lognormal distributions
- Task is to fit income distributions for 159 countries x 50 years x 4 functional forms – a total of 31,800 income distributions

Steps involved

Necessary steps for provision of available information

- **Choice of data for estimating the distributions.**
- **Estimation of the distributions.**
- **Computing inequality measures from the estimated parameters of the distributions.**

Data

- Aggregated data in the form of decile shares of income taken from World Bank's POVCAL website.
- For country years where data was not available, interpolation or extrapolation of shares was necessary.
  - Interpolation for share s_{t+r} lying *r* years after first observable point s_t with endpoint s_{t+k} :

$$s_{t+r} = \left(1 - \frac{r}{k}\right) s_t + \frac{r}{k} s_{t+k}$$

  - Extrapolation: needed when data for beginning or ending years (1970 and 2019) were not available. The values of the shares at the closest available year were maintained for the unavailable years, for up to five years.

Data - continued

Mean income: real per capita income in current US dollars obtained by converting per capita incomes in local currency units using the PPP exchange rates from module 1. Available in every year.

Notation for available data

population share for the *i*-th group (decile data) $c_i = 0.1$

income share s_i

mean income $\bar{y}$

mean income for *i*-th group $\bar{y}_i = \frac{s_i \bar{y}}{c_i}$

Generalized Method of Moments Estimation

Use data on c_i and $\bar{y}_i$ to estimate parameters θ of income distribution with pdf , $f(y;\theta)$ as well as the group bounds $z' = (z_1, z_2, \ldots, z_{N\text{-}1})$

The estimation methodology finds the values of θ and z that minimize the weighted squared difference between all the c_i and $\text{plim}\, c_i$, and $\bar{y}_i$ and all the $\text{plim}\, \bar{y}_i$

For a given income distribution, the plim's are known functions of θ and z.

cdf: $$F(z;\theta) = \int_0^z f(y)dy$$

first moment distribution fn: $$F^{(1)}(z;\theta) = \frac{1}{\mu}\int_0^z y f(y)dy$$

Generalized Method of Moments Estimation

$$\text{plim } c_i = F(z_i;\theta) - F(z_{i\text{-}1};\theta)$$

$$\text{plim } \overline{y}_i = \frac{\mu\left[F^{(1)}(z_i;\theta) - F^{(1)}(z_{i\text{-}1};\theta)\right]}{F(z_i;\theta) - F(z_{i\text{-}1};\theta)}$$

Minimize $$\varphi = \begin{pmatrix} c' - \text{plim } c' & \overline{y}' - \text{plim } \overline{y}' \end{pmatrix} W \begin{pmatrix} c - \text{plim } c \\ \overline{y} - \text{plim } \overline{y} \end{pmatrix}$$

Weight matrix W depends on θ and z via $F(z_i;\theta)$, $F^{(1)}(z_i;\theta)$ and the 2nd moment distribution function $F^{(2)}(z;\theta) = \frac{1}{\mu^{(2)}}\int_0^z y^2 f(y)dy$ which is also a known function of θ and z.

Minimize φ using an iterative process that updates W on each iteration.

Series available from UQICD V3.0

- Parameter estimates and their standard errors for each of the four distributions.

- Plots of figures for each of the distributions for years 1970, 1980, 1990, 2000, 2010 and 2019.

- Inequality measures computed from each distribution.
  - Gini coefficient
  - Theil coefficient
  - Income share of poorest 10%
  - Income share of poorest 30%
  - Income share of richest 10%
  - Income share of richest 1%

Regional and Global Growth and Inflation

“Global growth is forecast to slow from 6.0 percent in 2021 to 3.2 percent in 2022 and 2.7 percent in 2023…….. Global inflation is forecast to rise from 4.7 percent in 2021 to 8.8 percent in 2022 but to decline to 6.5 percent in 2023 and to 4.1 percent by 2024.”

World Economic Outlook 2022, IMF

Some questions about the IMF figures

- How is global growth computed? From WEO, this is a weighted average of growth rates in different countries with their shares in PPP terms as weights.
- What does 6.0 percent represent?
- What are the weights used?
- The approach is somewhat intuitive but without any formal framework.

UQICD Approach to Global Growth and Inflation

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- Global growth – growth in the size of the world economy

- What is the size of the world economy?
  - In PPP terms this is the sum of real GDP of all the countries in the world – In UQICD this is the sum of RGDP of all the countries covered.
  - $RGDP_W$ can be computed for each year, 1970 to 2019

- What are the components that make up the observed changes in $RGDP_W$ over time? UQICD decomposes this change into three components:
  - Growth – based on observed domestic growth in different countries
  - Inflation – measure based on observed inflation in different countries
  - PPP effect – change in $RGDP_W$ induced by changes in PPPs.

UQICD Approach to Global Growth and Inflation

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Size of the world economy in the years 1970 and 2019 from UQICD:

$RGDP_{W,1970}$ = **3.428 trillion US dollars**

$RGDP_{W,2019}$ = **125.929 trillion US dollars**

Ratio = $RGDP_{W,2019} / RGDP_{W,1970} = 36.73$

What part of this change can be attributed to: growth; inflation in countries around the world; changes in PPPs?

UQICD measures these three components using index number decomposition.

UQICD Approach to Global Growth and Inflation

It is a natural extension of the national level practice of comparing GDP at constant prices.

$$\frac{GDP_t}{GDP_{t-1}} = \frac{GDP_t / Def_{b,t}}{GDP_{t-1} / Def_{b,t-1}} \times \frac{Def_{b,t}}{Def_{b,t-1}} = \frac{CGDP_t}{CGDP_{t-1}} \times \frac{Def_{b,t}}{Def_{b,t-1}}$$

$$= \text{Growth} \times \text{inflation}$$

Now, we look at a decomposition of the ratio of RGDP of a country, *j*, in periods *t* and *t-1*

$$\frac{RGDP_{j,t}}{RGDP_{j,t-1}} = \frac{GDP_{j,t} / PPP_{j,t}}{GDP_{j,t-1} / PPP_{j,t-1}} = \frac{GDP_{j,t} / Def_{j,b,t}}{GDP_{j,t-1} / Def_{j,b,t-1}} \times \frac{Def_{j,b,t}}{Def_{j,b,t-1}} \times \frac{PPP_{j,t-1}}{PPP_{j,t}}$$

$$= \frac{CGDP_{j,t}}{CGDP_{j,t-1}} \times \frac{Def_{j,b,t}}{Def_{j,b,t-1}} \times \frac{PPP_{j,t-1}}{PPP_{j,t}}$$

$$= \text{GR}_{jt-1,t} \times \text{inflation}_{j,t-t,t} \times \text{PPP change}_{j,t-1,t} = \text{GR}_{\text{j}} \times \text{Price effect}_{j,t-1,t}$$

UQICD Approach to Global Growth and Inflation

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In UQICD, we make use of the algebra of the Sato-Vartia index to derive a decomposition of the ratio of RGDP of the world in periods *t* and *t-1* into different components.

$$\frac{RGDP_{W,t}}{RGDP_{W,t-1}} = \frac{\sum_{j=1}^{M} RGDP_{j,t}}{\sum_{j=1}^{M} RGDP_{j,t-1}} = \prod_{j=1}^{M}\left[\frac{CGDP_{j,t}}{CGDP_{j,t-1}}\right]^{\omega_j} \prod_{j=1}^{M}\left[\frac{Def_{j,b,t}}{Def_{j,b,t-1}}\right]^{\omega_j} \prod_{j=1}^{M}\left[\frac{PPP_{j,t-1}}{PPP_{j,t}}\right]^{\omega_j}$$

$$= \prod_{j=1}^{M}\left[Dom.GR_{j,t-1,t}\right]^{\omega_j} \prod_{j=1}^{M}\left[Dom.Inf_{j,t-t,t}\right]^{\omega_j} \prod_{j=1}^{M}\left[PPP\,effect\right]^{\omega_j}$$

$$= Global\ growth \times Average\ Dom\ Inf \times PPP\,change\,effect$$

$$= Global\ growth \times Global\ Inflation$$

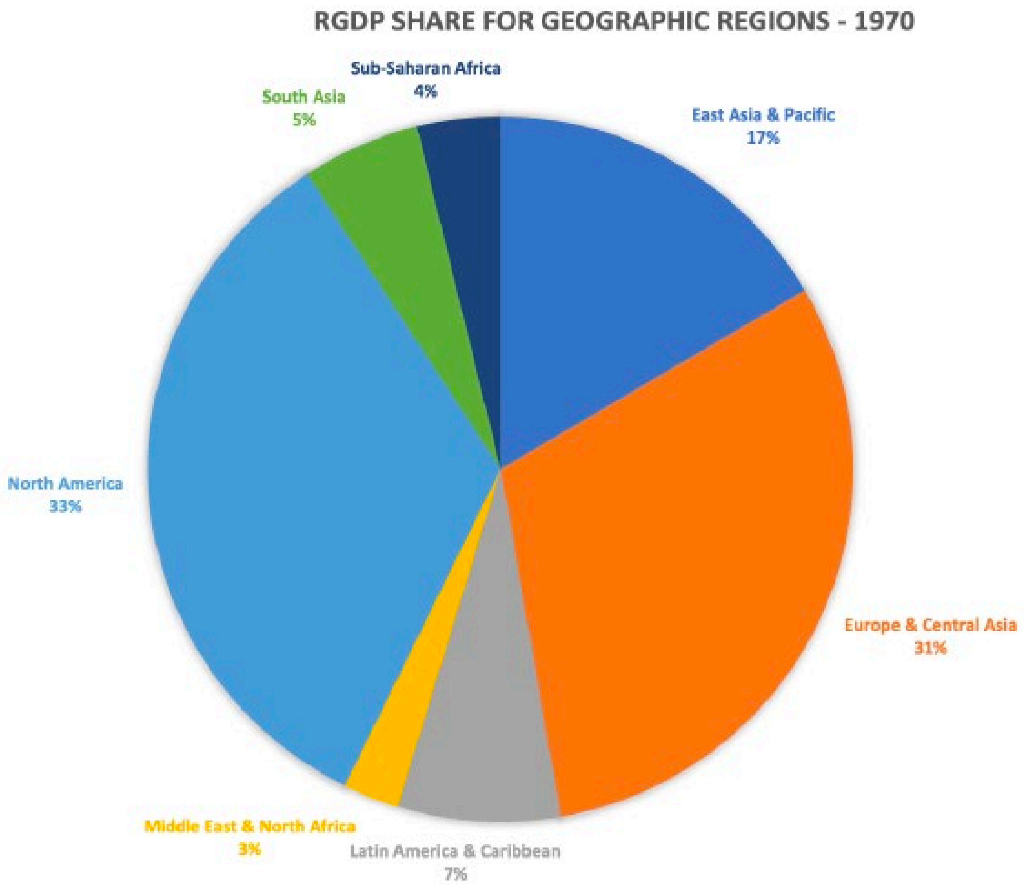
where the weights are the Sato-Vartia weights (see p.47 of the UQICD V3.0 User Guide)

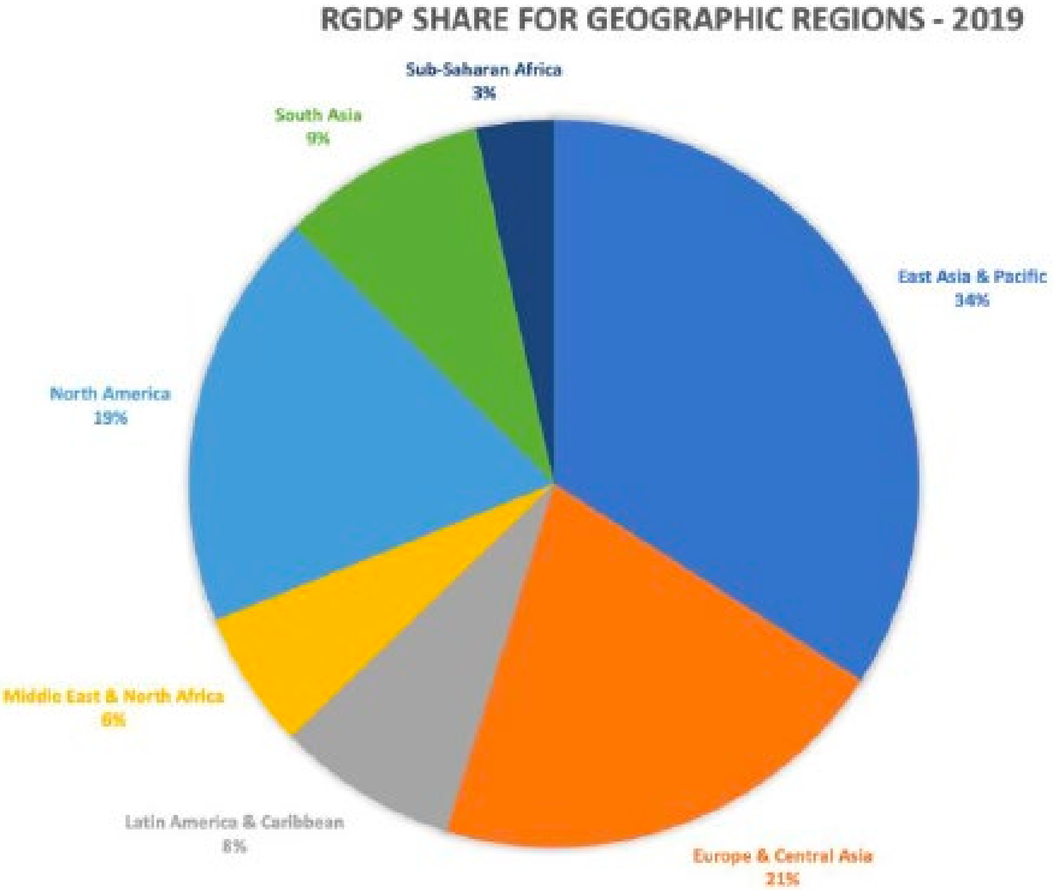
Trends in Economic Welfare Over the Last Fifty Years

World economy: last five decades

	WORLD				
	1970	1980	1990	2000	2019
Population (bill)	3.374	4.096	5.263	6.069	7.619
RGDP ($ trillion)	3.428	11.715	24.823	43.222	133.088
NGDP ($ trillion)	2.931	11.218	22.743	33.166	86.473
CGDP ($trillion)	20.397	30.495	45.893	61.163	118.187

Regional Shares





Share of RGDP of China, India, USA and Japan in the World Economy over the years

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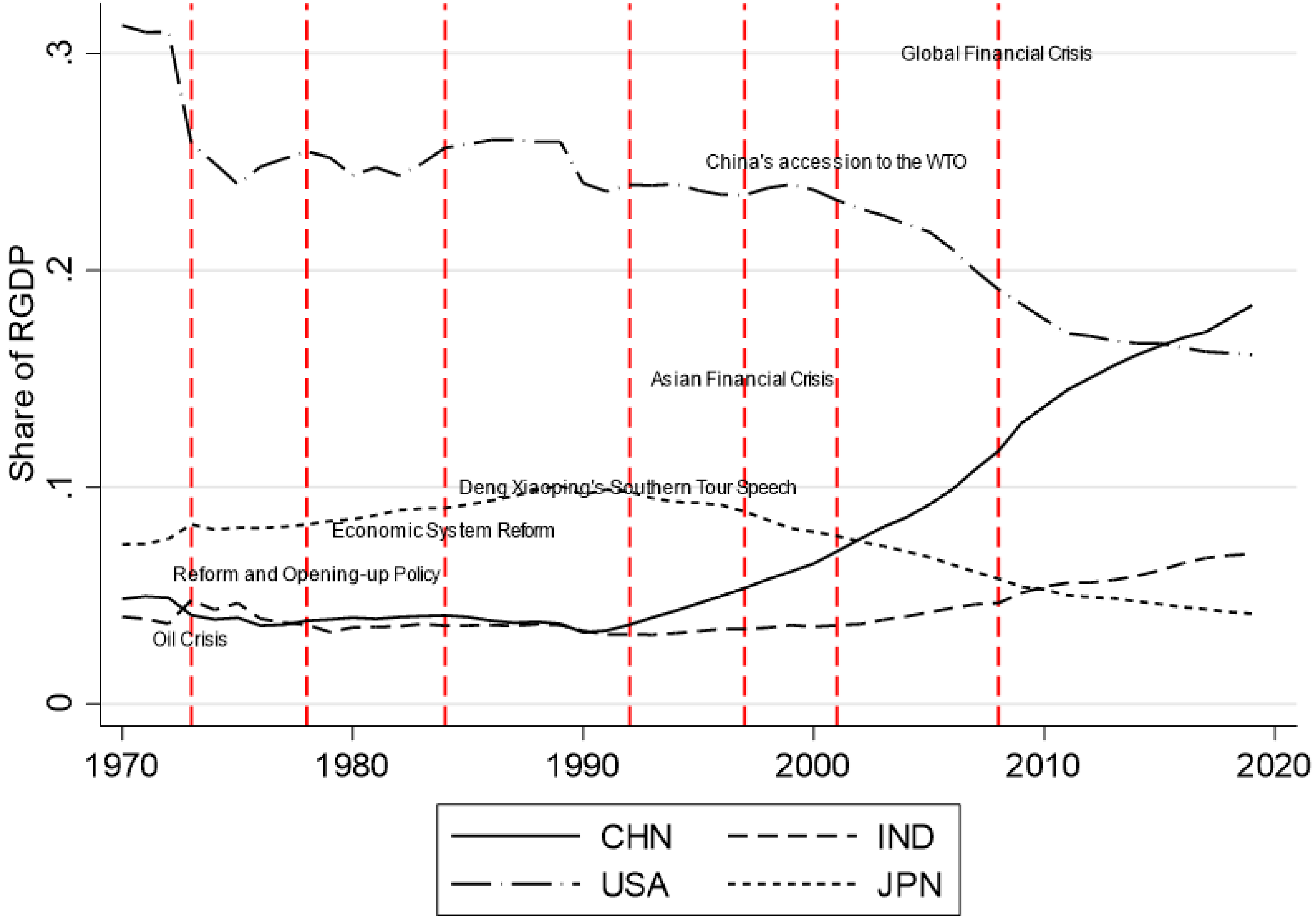
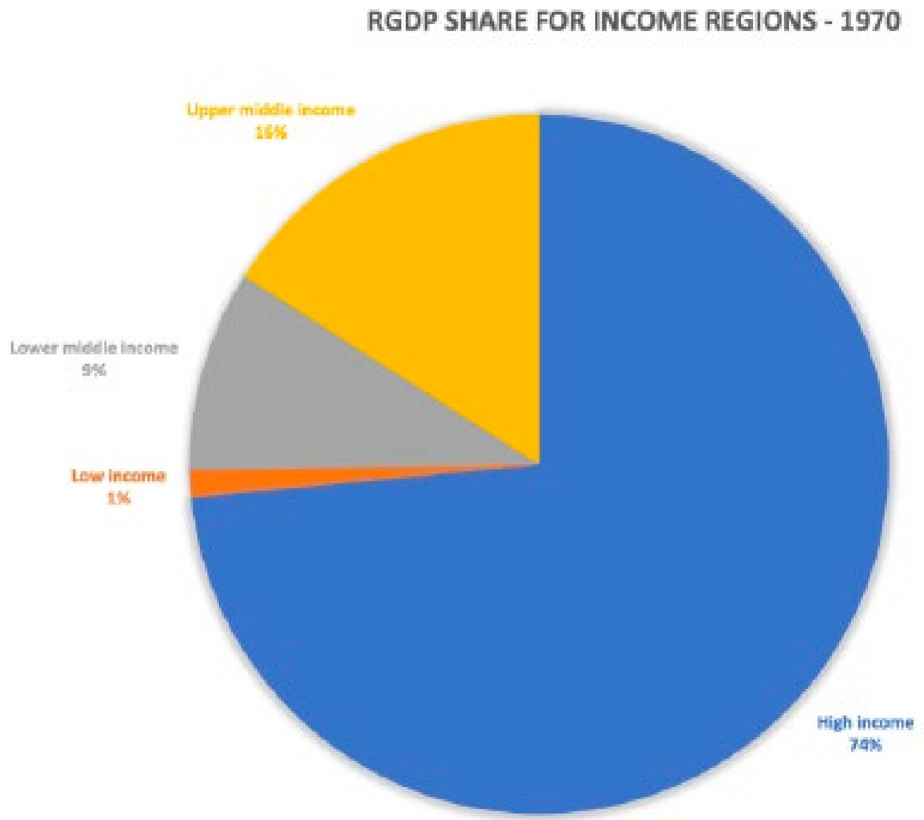
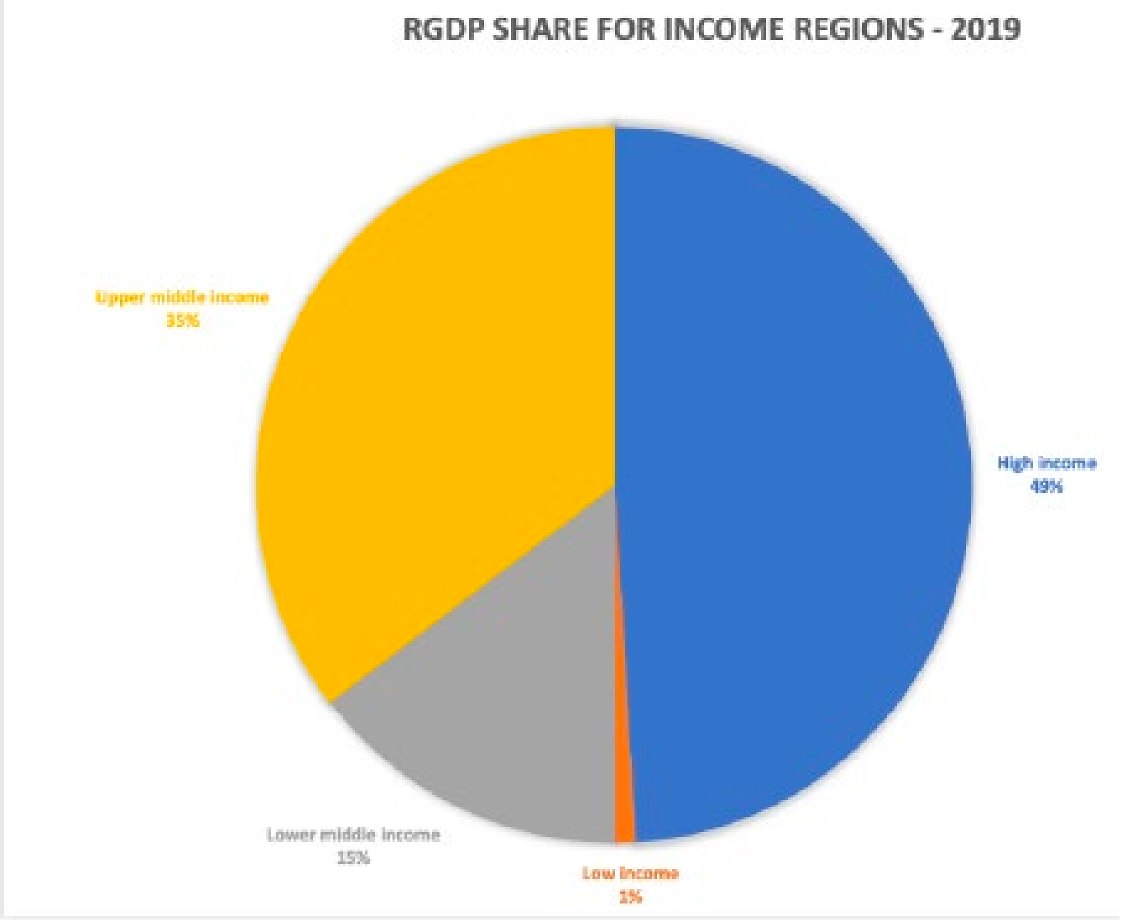
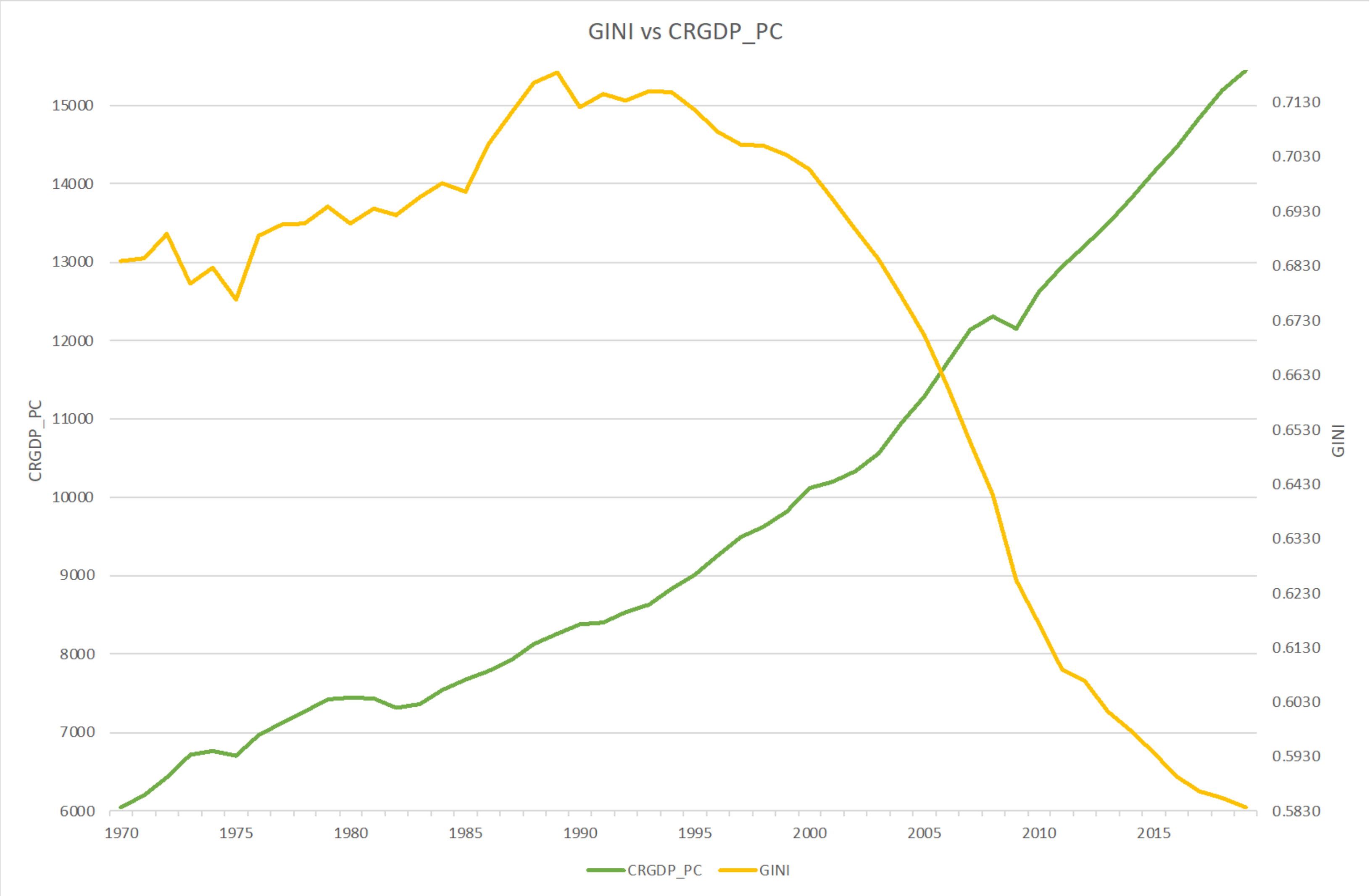


Shares of Income Groups





Global Growth and Inequality 1970-2019



Real per capita GDP (constant 2011 prices and USD) inequality and welfare - selected countries

Country	1970			2019		
	CRGDP_PC	Gini	Welfare	CRGDP_PC	Gini	Welfare
Australia	21361	0.316	14611	46156	0.3495	30024
China	446	0.2819	321	15916	0.3874	9750
Germany	18909	0.2917	13393	45612	0.3183	31094
Japan	14901	0.3479	9717	39237	0.3405	25877
India	1142	0.3199	777	6208	0.3535	4014
United Kingdom	17106	0.3009	11959	41177	0.3507	26736
United States	23188	0.3555	14944	56926	0.4154	33279

Real per capita GDP (constant 2011 prices and USD) Inequality and Welfare – by Regions

	1970			2000			2019		
Region	CRGDP_PC	Gini	Welfare	CRGDP_PC	Gini	Welfare	CRGDP_PC	Gini	Welfare
East Asia & Pacific	2412	0.6157	927	7108	0.6414	2549	17893	0.4464	9905
Europe & Central Asia	15370	0.3609	9823	22160	0.5058	10952	30617	0.4264	17562
Latin America & Caribbean	8052	0.5929	3278	12059	0.5768	5103	14888	0.5135	7243
Middle East & North Africa	11617	0.4534	6350	15348	0.4957	7740	20388	0.4522	11169
North America	23014	0.3800	14269	44957	0.4031	26835	55694	0.4188	32369
South Asia	1251	0.3291	839	2395	0.3539	1547	5811	0.3694	3664
Sub-Saharan Africa	2954	0.5310	1386	2689	0.5161	1301	3928	0.4977	1973
WORLD	6308	0.6847	1989	10592	0.7006	3172	16396	0.5859	6790

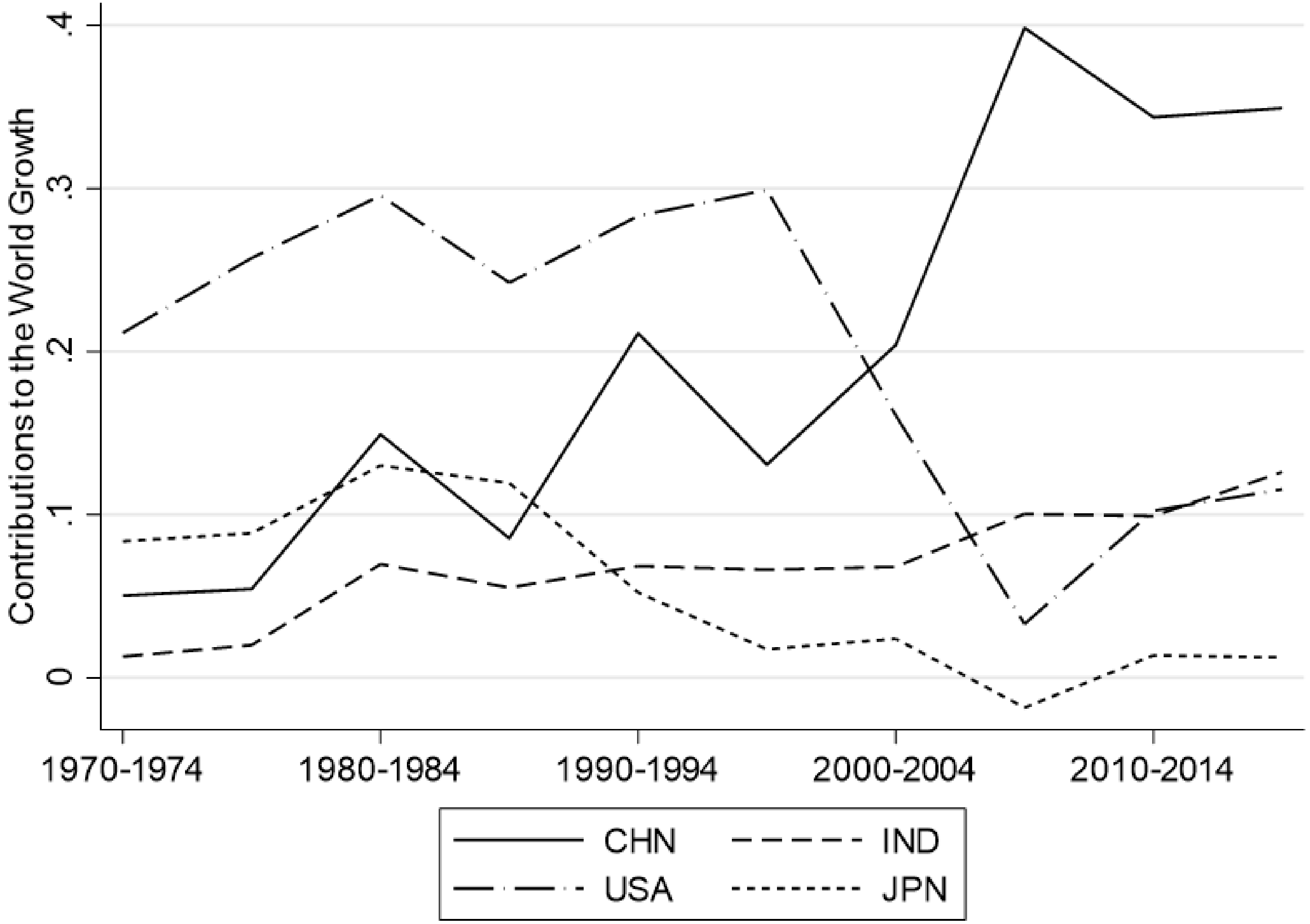
Global Growth and Inflation 1970-2019

We consider the change in RGDP of the world over the period 1970 to 2019

Measure	VALUE
RGDP_1970	$3.428 trillion
RGDP_2019	$125.929 trillion
RGDP_2019/RGDP_1970	36.73
DOM_GR	6.33
DOM_INF	61.64
PPP_EFFECT	0.10
PRICE_EFFECT	6.13

Ln(Growth rates)		% contribution
LN(RGDP ratio)	3.60	
LN (DOM_GR)	1.79	**49.68%**
Australia	0.02	0.98%
China	0.45	25.36%
Germany	0.05	2.62%
India	0.15	8.37%
Japan	0.07	3.90%
United States	0.33	18.51%

Contribution of China, India, USA and Japan to the world growth (five-year periods)



Global inequality

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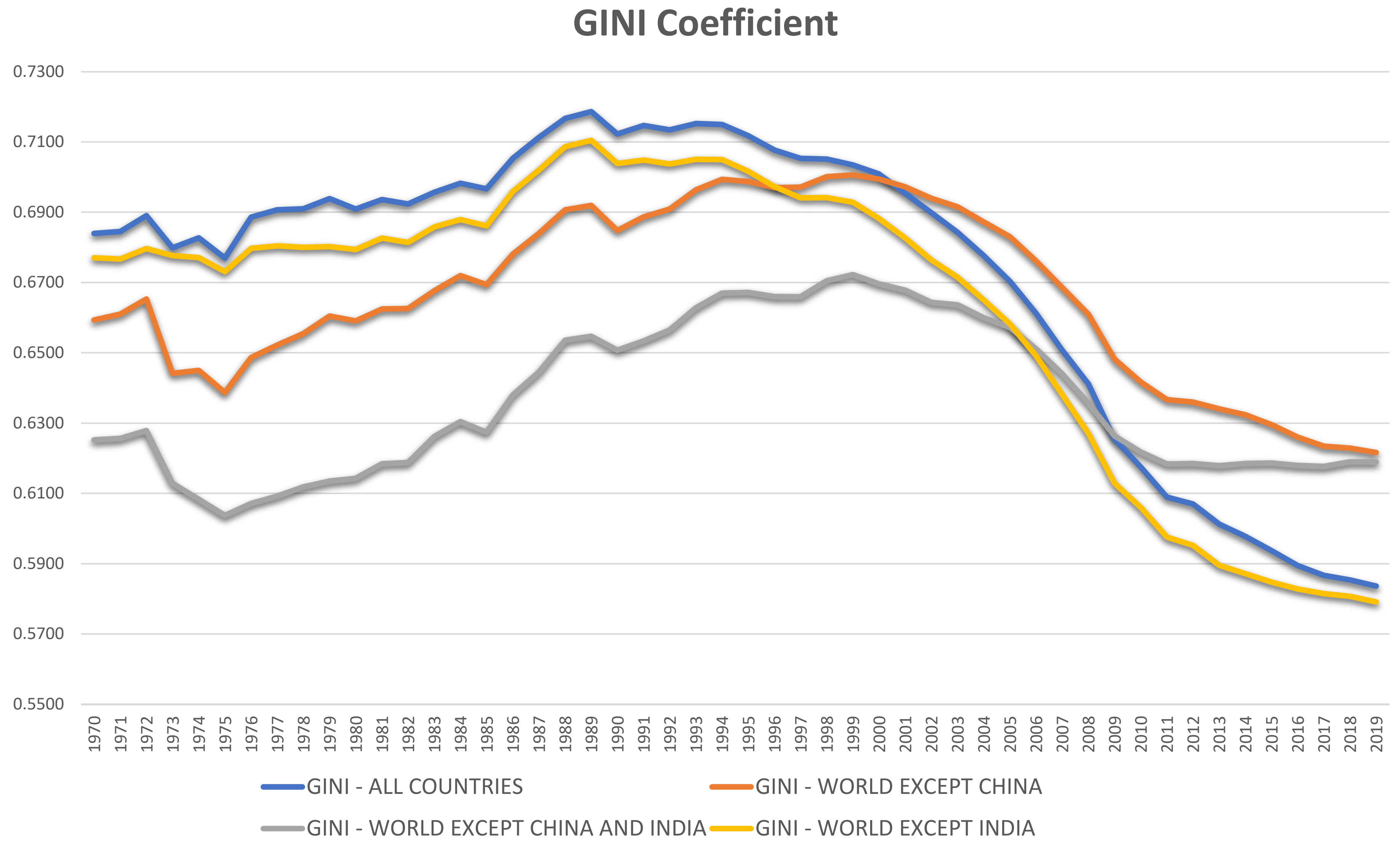
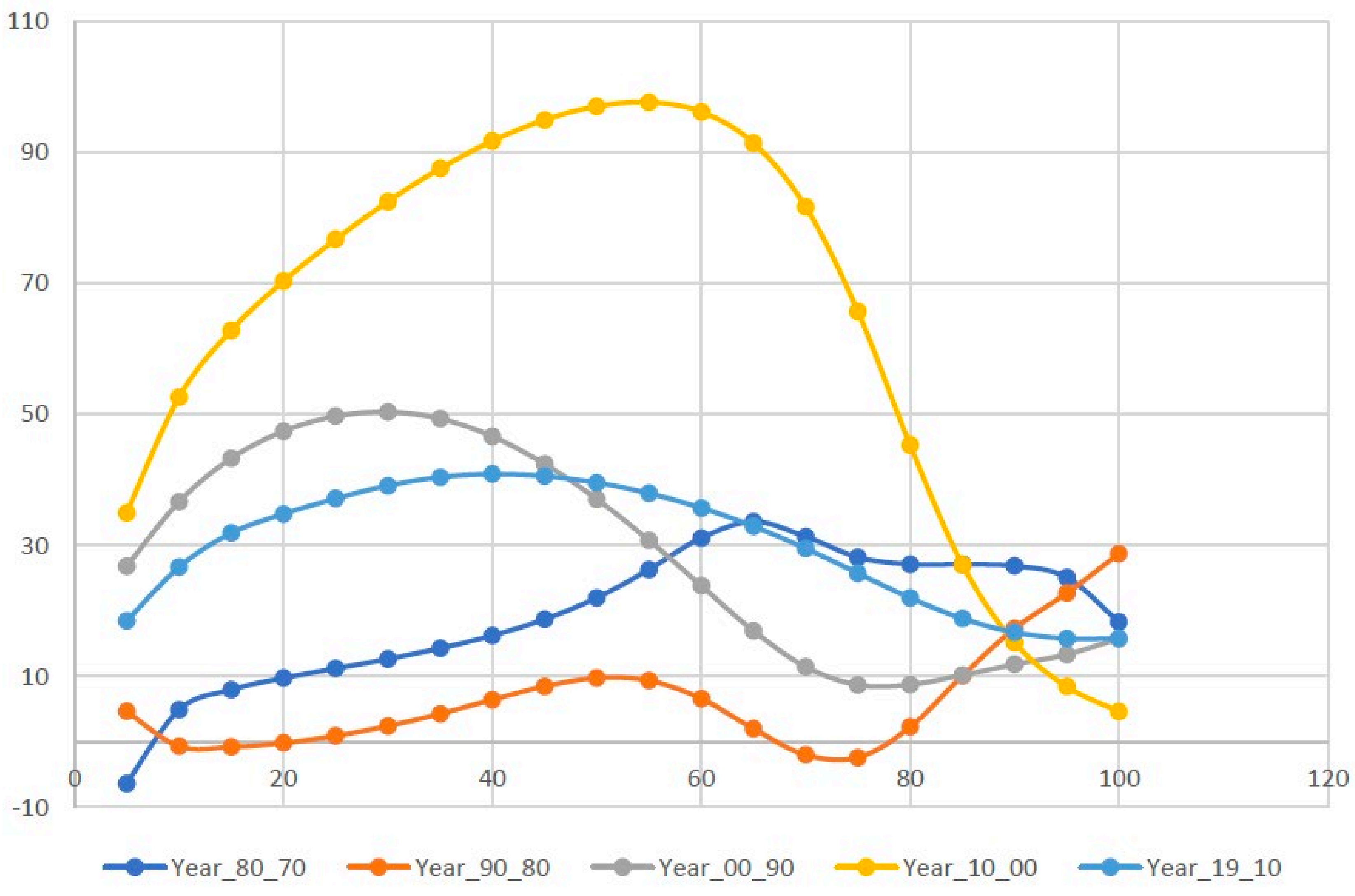
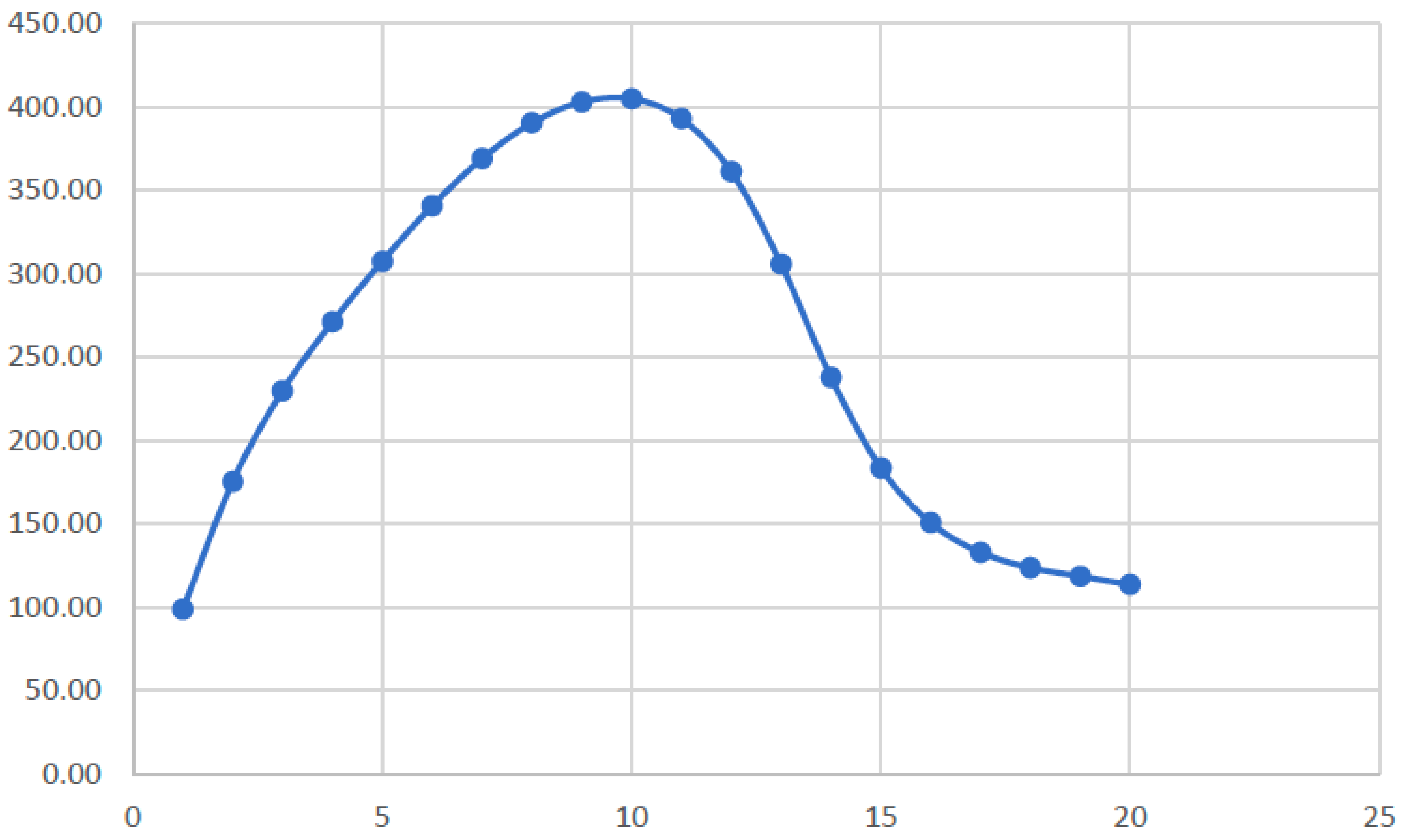


World Growth Incidence Curves – by decades



World Growth Incidence Curves – whole period



Income convergence

Global inequality

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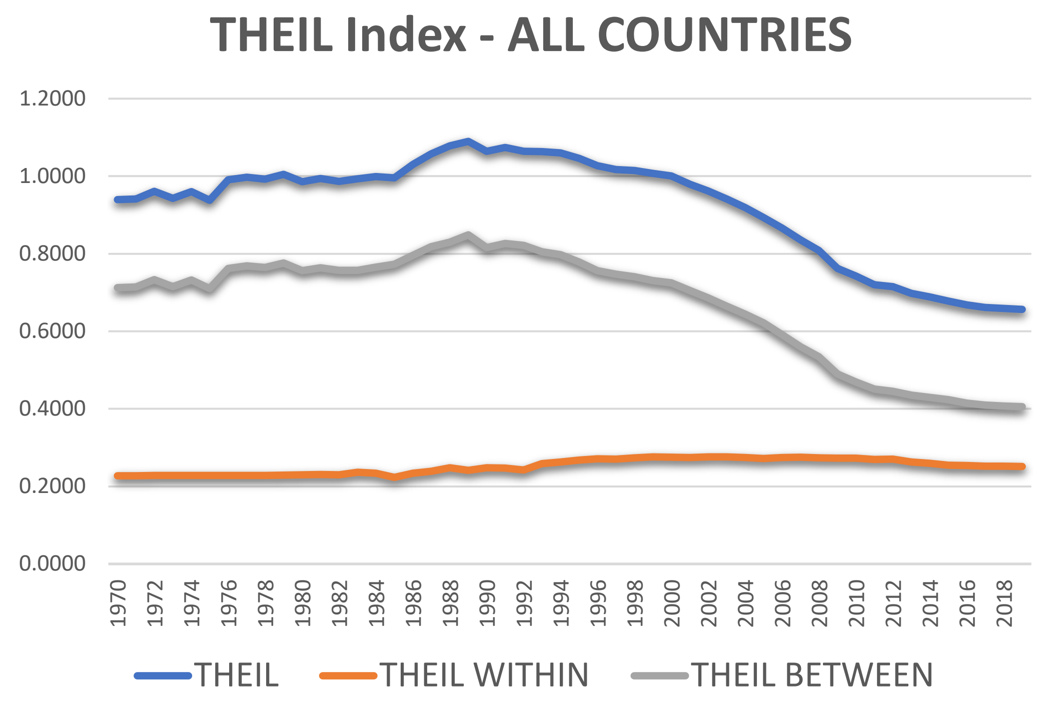
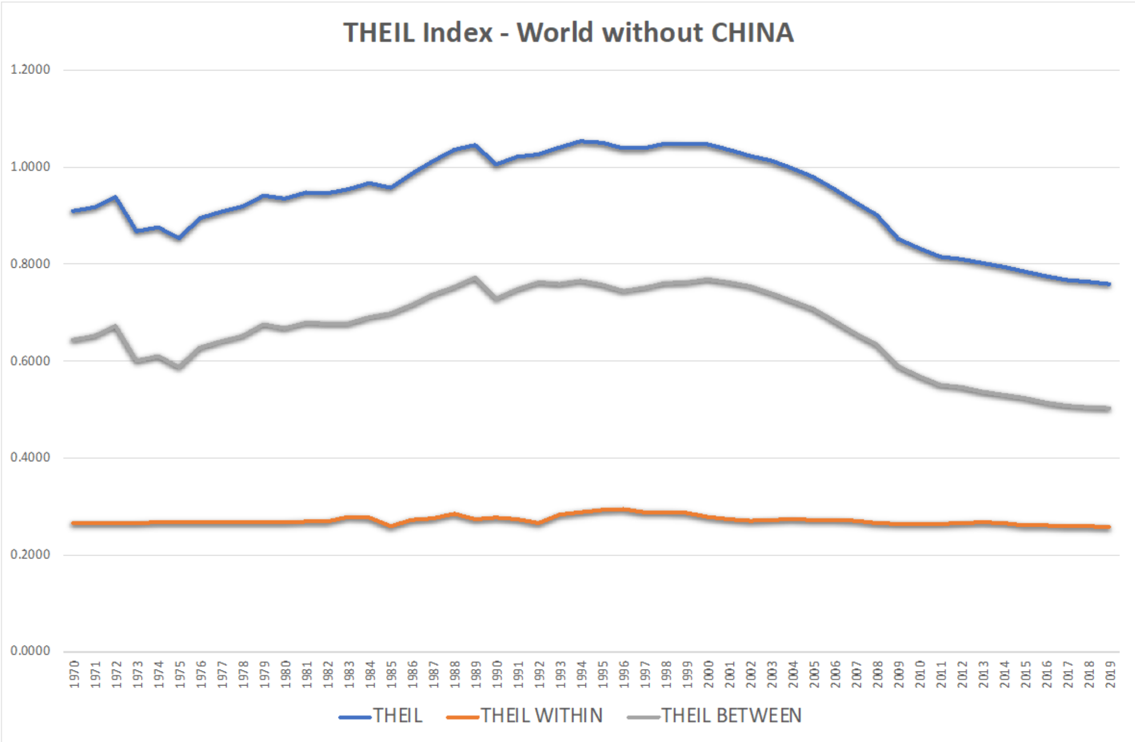
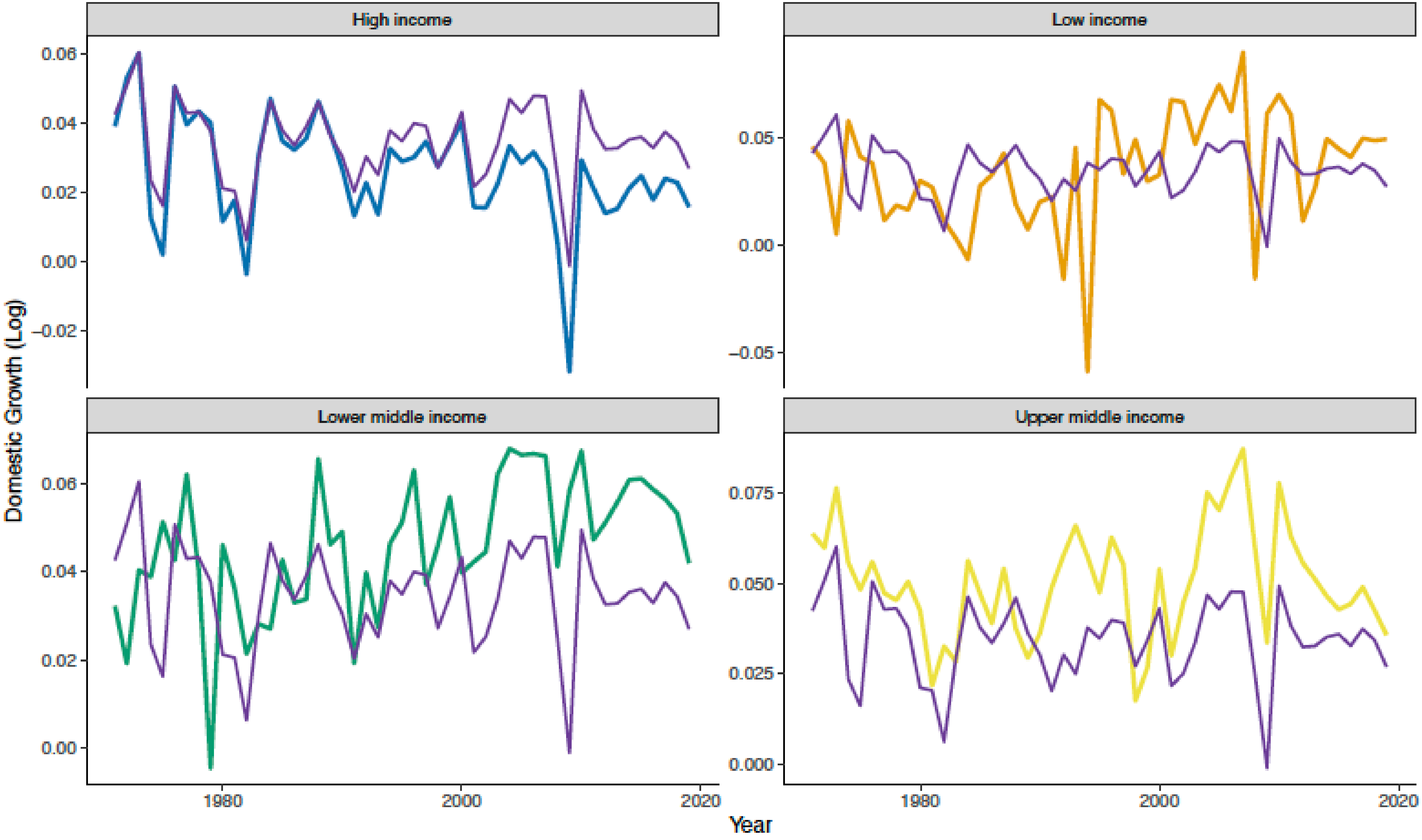




Growth: Income-based groups



Absolute convergence

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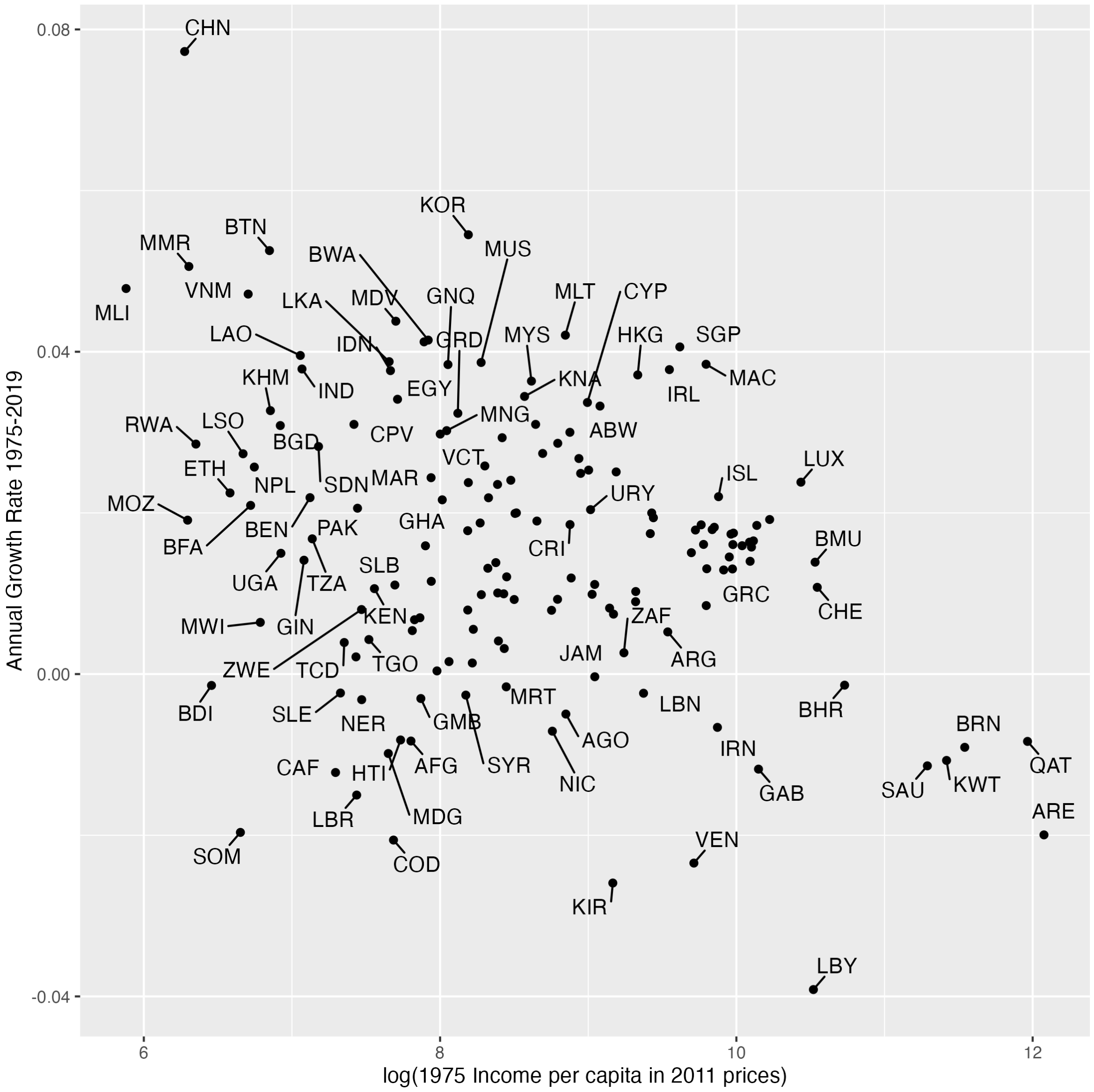
Absolute β-Convergence Regressions – (World)					
	1975-85	1985-1995	1995-2005	2005-19	1975-2019
Estimated β	0.002	-0.003	0.002	0.007	0.004
	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
N	159	159	159	159	159
R2	0.009	0.015	0.008	0.158	0.095
AIC	-634.9	-634.3	-699.5	-769.3	-847.2
BIC	-625.7	-625.1	-690.3	-760.0	-838.0
Log.Lik.	320.44	320.16	352.75	387.63	426.61
F	1.392	2.319	1.261	29.471	16.524
RMSE	0.03	0.03	0.03	0.02	0.02

Regression: y= a – βx; if β > 0 the data exhibits absolute convergence (Sala-i-Martin, 1996, Econ Journal)

$y = (1/(T-t))*\ln(CRGDP\_PC_T/CRGDP\_PC_t)$; $x = \ln(CRGDP\_PC_t)$;

Note: Adjusted R2 for absolute convergence regression for 110 countries and 1960-1990 reported in Sala-i-Martin (Economic Journal, 1996) was 0.04!

Absolute convergence



Absolute convergence – with World Bank Income Group Fixed Effects

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Absolute β-Convergence Regressions – (World)

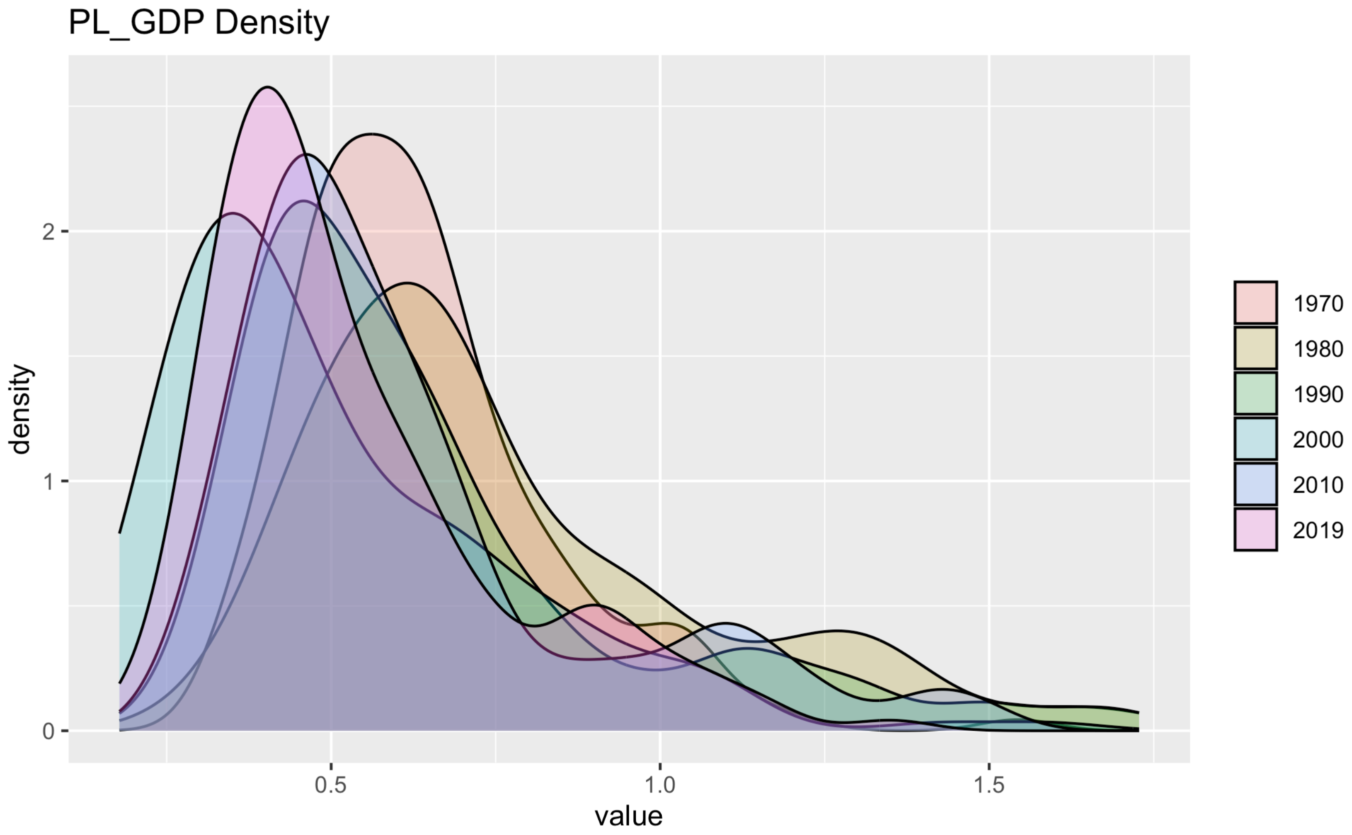
	1975-85	1985-1995	1995-2005	2005-19	1975-2019
Estimated β	0.025	0.022	0.020	0.021	0.019
	(0.003)	(0.004)	(0.004)	(0.003)	(0.001)
N	159	159	159	159	159
R2	0.434	0.333	0.177	0.319	0.702
AIC	-717.9	-690.3	-723.2	-797.0	-1017.9
BIC	-699.5	-671.9	-704.8	-778.5	-999.5
Log.Lik.	364.966	351.147	367.590	404.477	514.951
F	29.508	19.193	8.272	18.021	90.774
RMSE	0.02	0.03	0.02	0.02	0.01

Regression: y= a – βx + WBIncome FE; if β > 0 the data exhibits absolute convergence (Sala-i-Martin, 1996, Econ Journal)

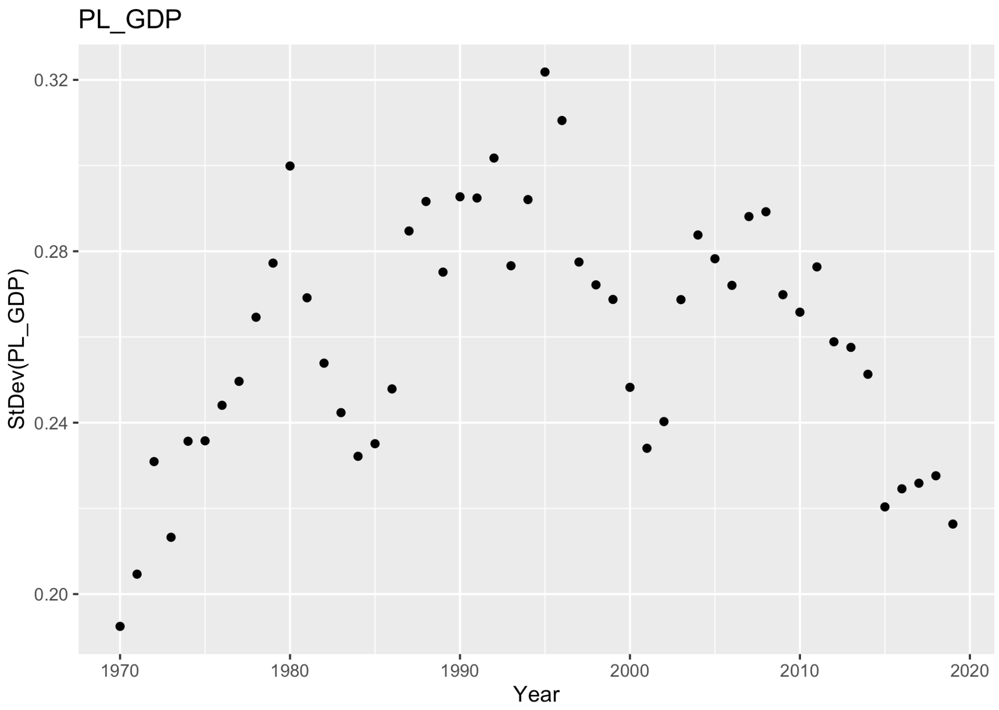
$y = (1/(T-t))*\ln(CRGDP\_PC_T/CRGDP\_PC_t)$; $x = \ln(CRGDP\_PC_t)$;

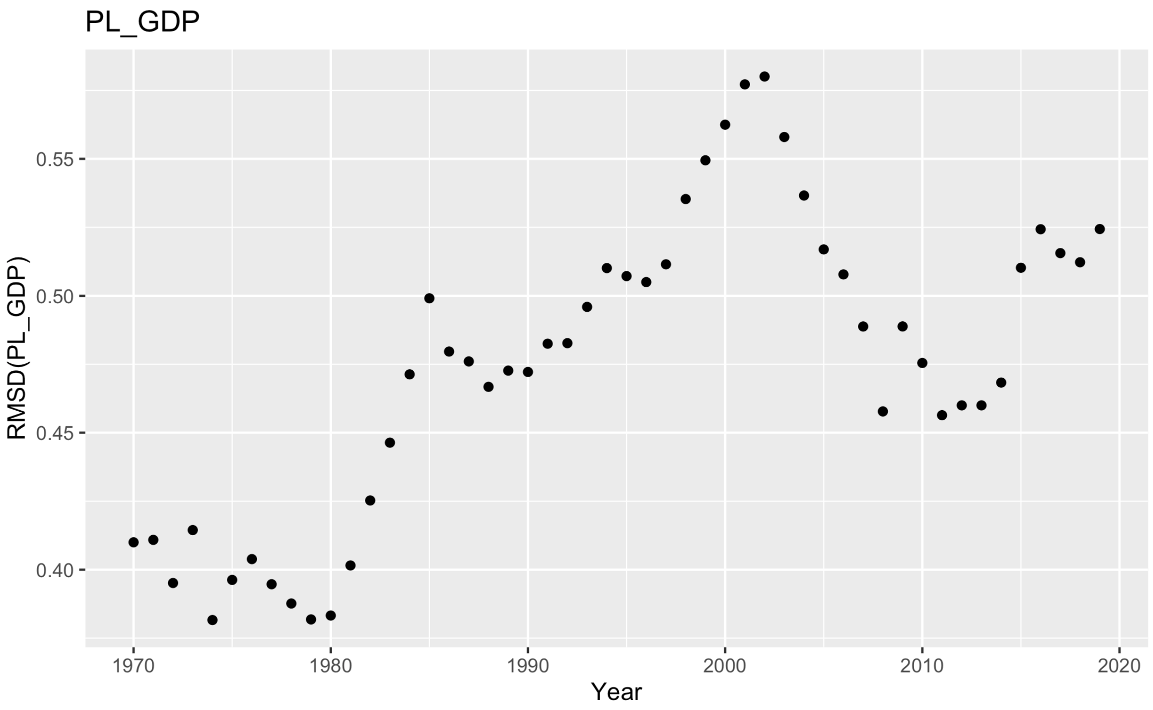
Price Level Convergence

Price levels

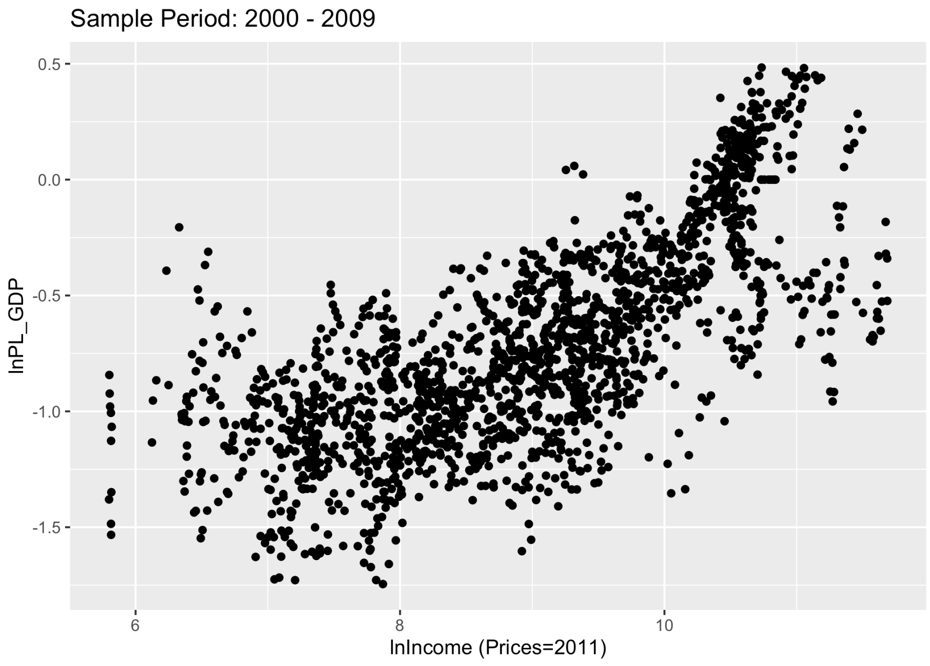


Price levels



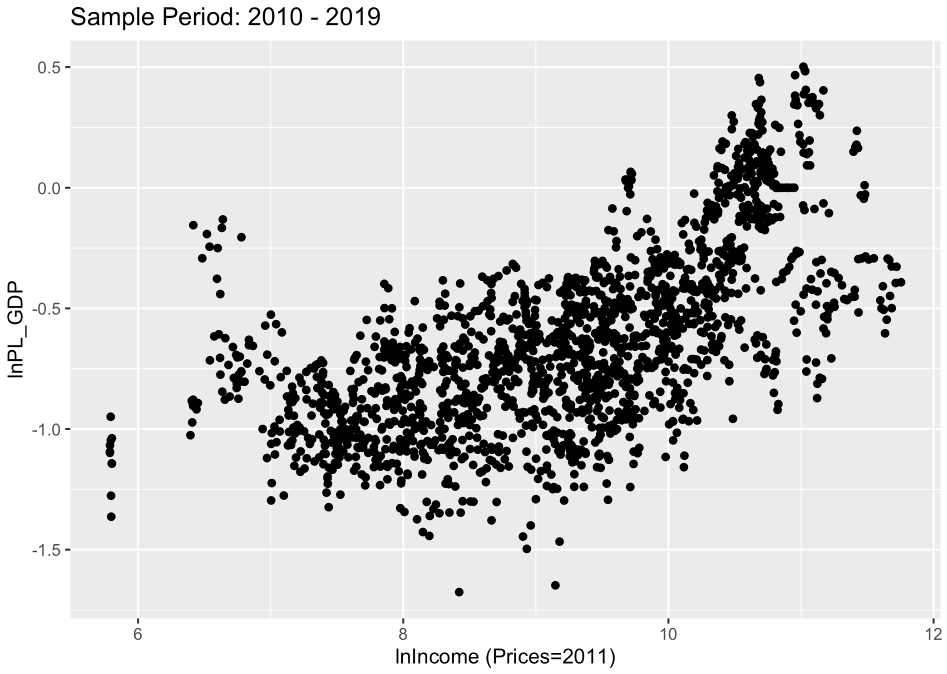


Price levels



	Linear 2000-2009		Quadratic 2000-2009	
Predictors	*Estimates*	*p*	*Estimates*	*p*
lnIncome	0.25	**<0.001**	-0.62	**<0.001**
lnIncome2			0.05	**<0.001**
Observations	1824		1824	
R^2 / R^2 adjusted	0.875 / 0.875		0.886 / 0.885	

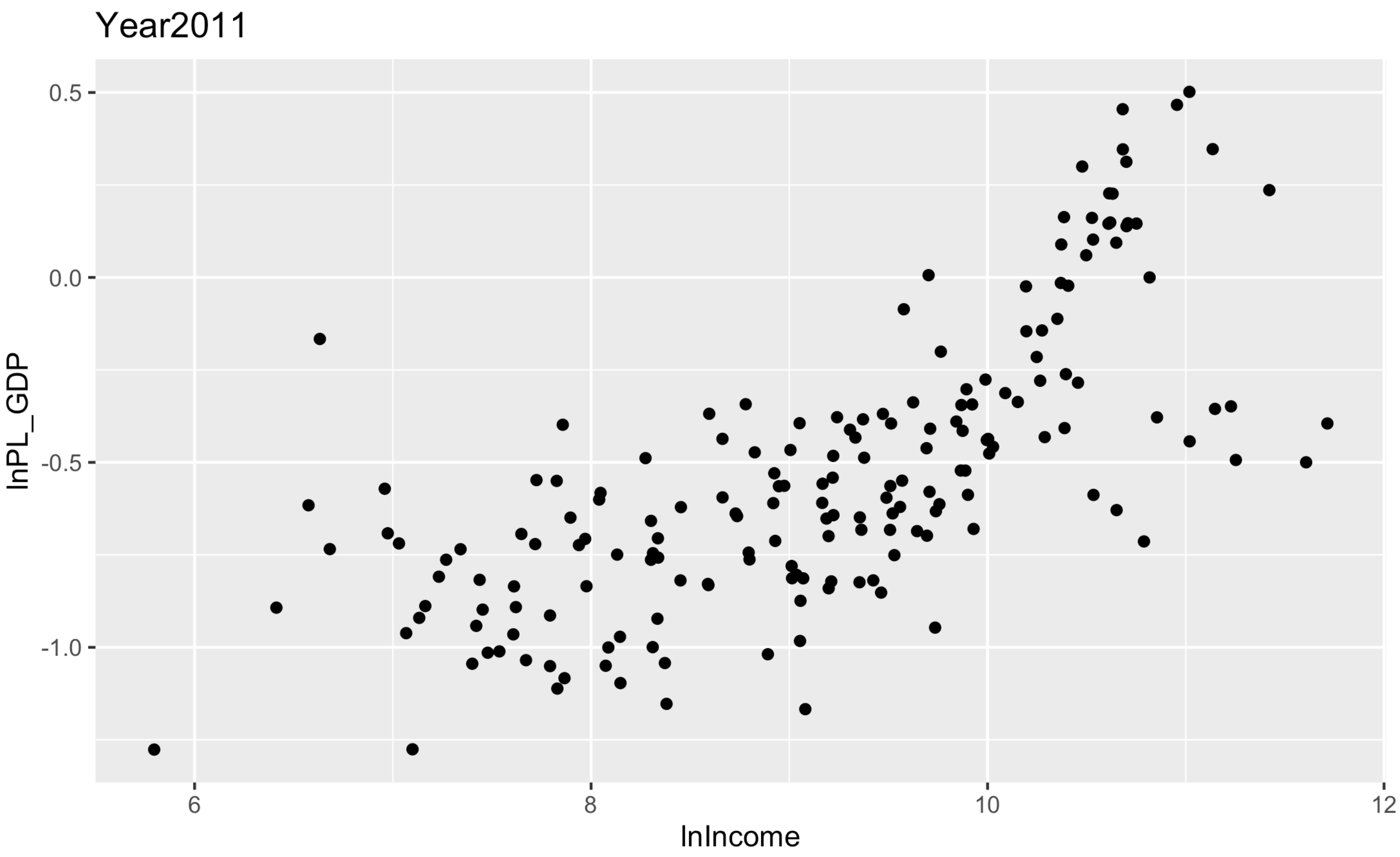
Estimated turning point: $490



	Linear 2010-2019		Quadratic 2010-2019	
Predictors	*Estimates*	*p*	*Estimates*	*p*
lnIncome	0.21	**<0.001**	-0.84	**<0.001**
lnIncome2			0.06	**<0.001**
Observations	1830		1830	
R^2 / R^2 adjusted	0.858 / 0.857		0.876 / 0.875	

Estimated turning point: $1095

Price levels – scatter diagram



Price Level Regressions – Penn Effect

GDP Price Level Regressions (balanced panels)												
	2000-09				2010-2019				1970-2019			
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
lnIncome	0.25 (0.001)	-0.62 (0.001)	0.15 (0.001)	-0.10 (0.572)	0.21 (0.001)	-0.84 (0.001)	-0.04 (0.062)	0.41 (0.019)	0.22 (0.001)	-0.35 (0.001)	0.15 (0.001)	-0.64 (0.001)
lnIncome^2		0.05 (0.001)		0.01 (0.143)		0.05 (0.001)		-0.02 (0.010)		0.03 (0.001)		0.05 (0.001)
Year FE	X	X	X	X	X	X	X	X	X	X	X	X
Country FE			X	X			X	X			X	X
N	1824	1824	1824	1824	1830	1830	1830	1830	7950	7950	7950	7950
R^2 / R^2 adjusted	0.875 / 0.875	0.886 / 0.885	0.983 / 0.981	0.983 / 0.981	0.858 / 0.857	0.876 / 0.875	0.986 / 0.985	0.986 / 0.985	0.935 / 0.933	0.842 / 0.841	0.849 / 0.848	0.938 / 0.936

Notes: p-values in brackets

Convergence of Income Distributions

Divergence of Income Distributions

Here we wish to examine:

- **If income distributions of countries are diverging from period to period**
- **If income distributions of the world are bunching together (convergence)**

- **We use the approach developed by Hajargasht (2022) to derive these measures for specific distributions.**

- **The approach relies on the Kullback-Leibler measure divergence between two distributions:**

Divergence of Income Distributions

Kullback-Leibler (KL) divergence Measure:

$$KL(f,g)=\int_0^\infty \ln\left(f(x)/g(x)\right)f(x)dx$$

where $f(x)$ and $g(x)$ are PDFs of the two distributions. This measure depends on the scale of the variable x.

A scale free alternative to KL due to Cowell et al. 2009 in income share

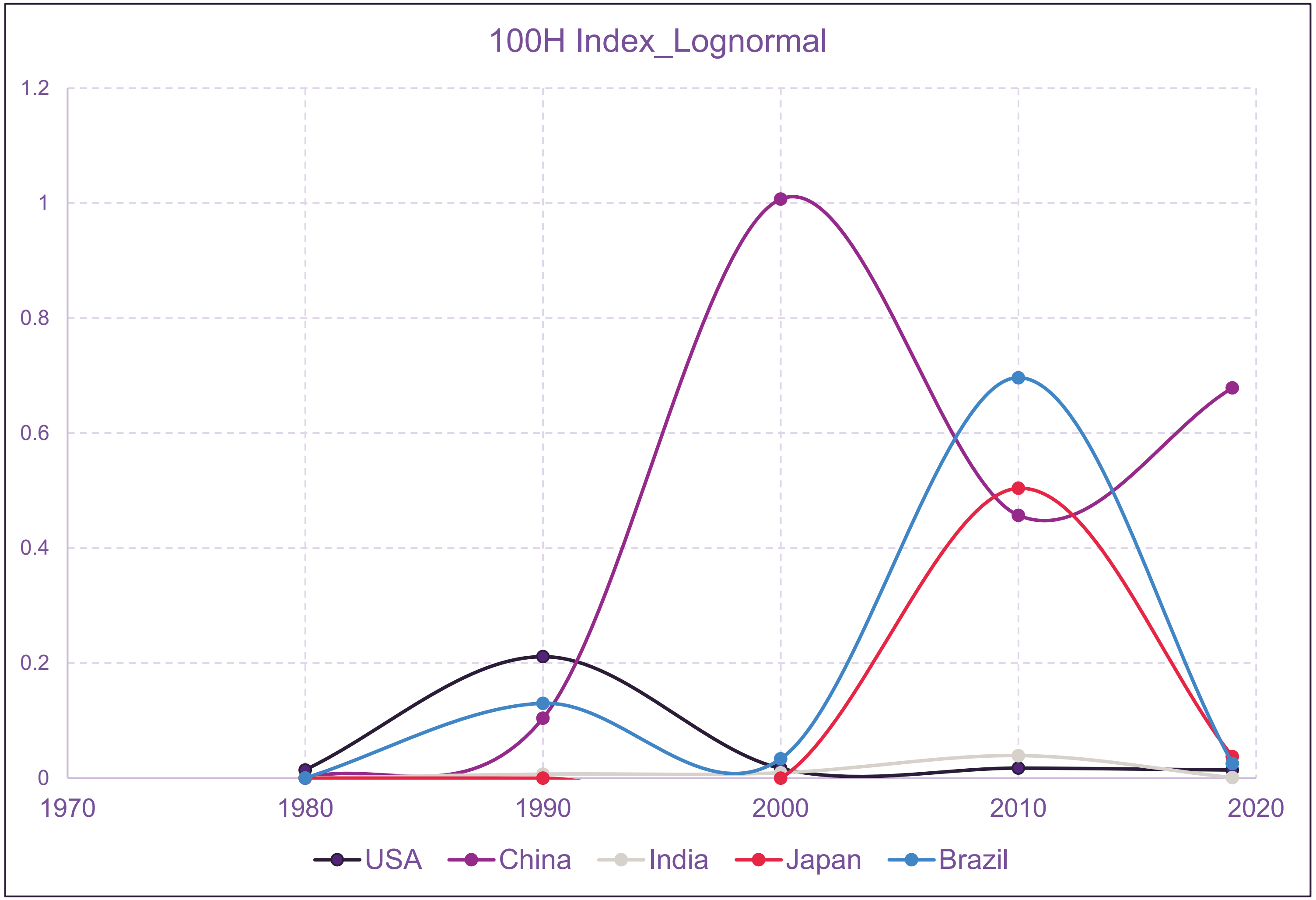
$$KL(f_1,f_2)=H(s_1,s_2)=\int_0^1 \ln\left(s_1(\pi)/s_2(\pi)\right)s_1(\pi)d\pi$$

where: π denotes population size normalised to [0,1];

$s(\pi)$ is infinitesimal share of income;

$s(\pi)=\dfrac{\partial\eta(\pi)}{\partial\pi}$ where $\eta(\pi)$ denotes the Lorenz curve

Convergence of Income Distributions



Measuring Total Divergence among of Income Distributions of countries in the world

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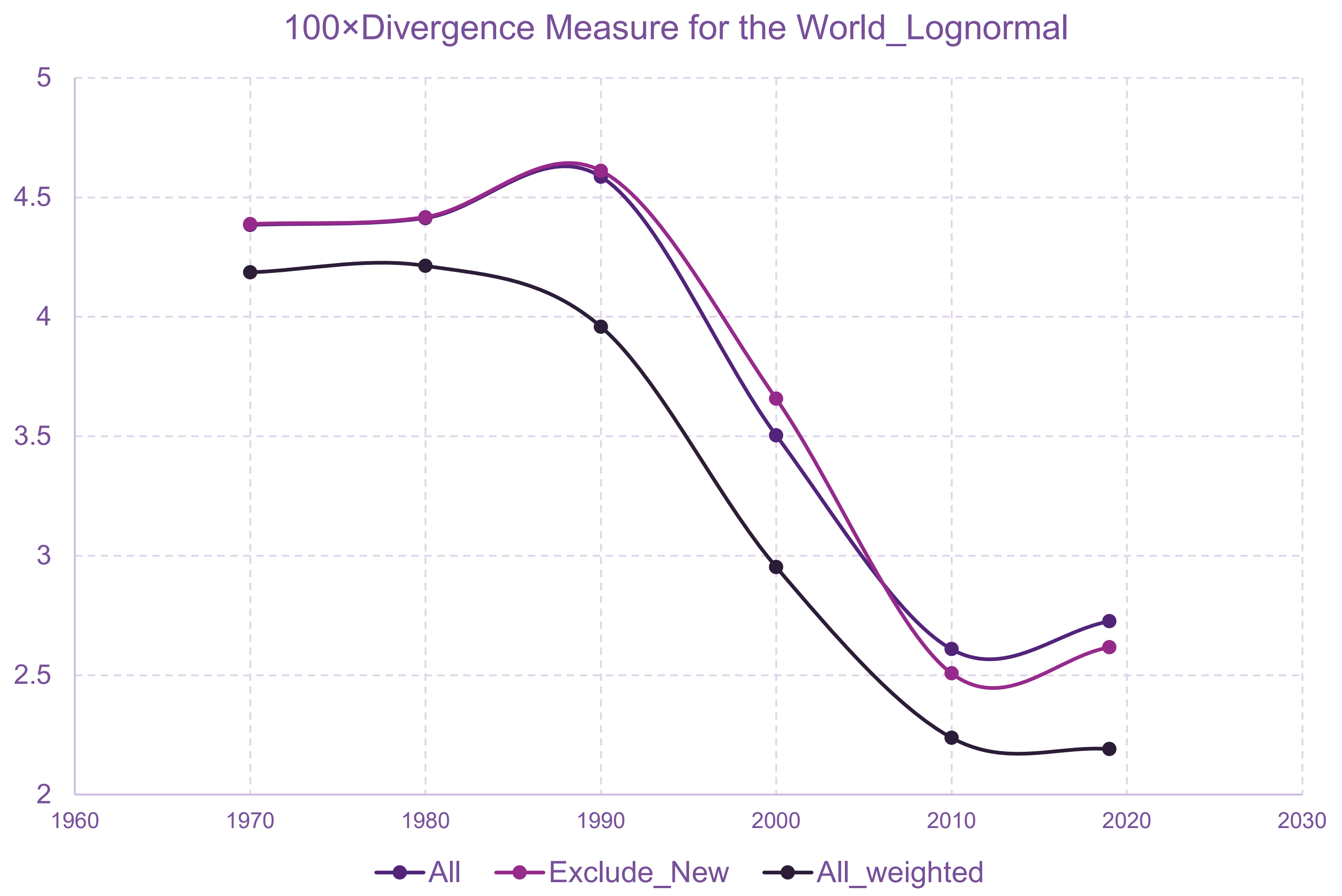
- **We measure divergence between distribution of country *j* from the distribution of country *k using the H-measure described - for all pairs j and* k.**
- **We propose a measure of total divergence as the average of the divergence between income distributions of all pairs of countries.**

$$unweighted: TD(f_1,\dots,f_M) = \frac{1}{M(M-1)}\sum_{j=1}^{M}\sum_{k=1}^{M} H(f_j, f_k)$$

Population share weighted:

$$TD^W(f_1,\dots,f_M) = \frac{1}{(M-1)}\sum_{j=1}^{M}\sum_{k=1}^{M}\left(\frac{w_j + w_k}{2}\right) H(f_j, f_k) \quad where\ w_j\ is\ pop\ share\ of\ country\ j$$

Convergence of Income Distributions



Total number of countries: 158 **Countries in existence since 1990: 25**

There is more work to report!

For now, thank you!

Choice of Income Distributions

UQICD offers estimated parameters and quantities of economic interest for four distributions.

- Lognormal distribution: a simple two parameter distribution, widely used. Not adequate to capture extended right tail, a characteristic of income distributions. Also implies symmetric Lorenz curves.
- Generalized Beta Type 2 distribution: a four-parameter distribution that is widely used, provides good fit for most distributions. There is some type of dependence in parameter space – this leads to large values of parameters in one instance and somewhat smaller values for other. However, it works fine when it comes to estimates of inequality.
- Mixture of lognormal distributions – flexible, can accommodate multimodal distributions. Found evidence of over parametrization with large values of standard errors for some parameters. Estimation had to be restricted two or three lognormal distributions with two of them having same variance.

Pareto-Lognormal Distribution

Of the four distribution series provided in UQICD, our recommendation/preference is the Pareto-Lognormal (PLN) distribution.

PLN is the product of two independent random variables that are distributed as lognormal and Pareto.

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right\} \qquad x > 0,\ -\infty < \mu < \infty,\ \sigma > 0$$

$$f(z) = \frac{\alpha}{z^{\alpha+1}} \qquad z > 1,\ \alpha > 0$$

If Y = X x Z, then density function of Y is given by

$$f(y) = \frac{\alpha}{y^{\alpha+1}} \exp\left\{\alpha\left(\mu + \frac{\alpha\sigma^2}{2}\right)\right\} \Phi\left(\frac{\ln y - \mu - \alpha\sigma^2}{\sigma}\right) \qquad y > 0$$

Pareto-Lognormal distribution

Note that as α becomes large, PLN tends towards lognormal distribution. Smaller values of α lead to distributions with fat tails.

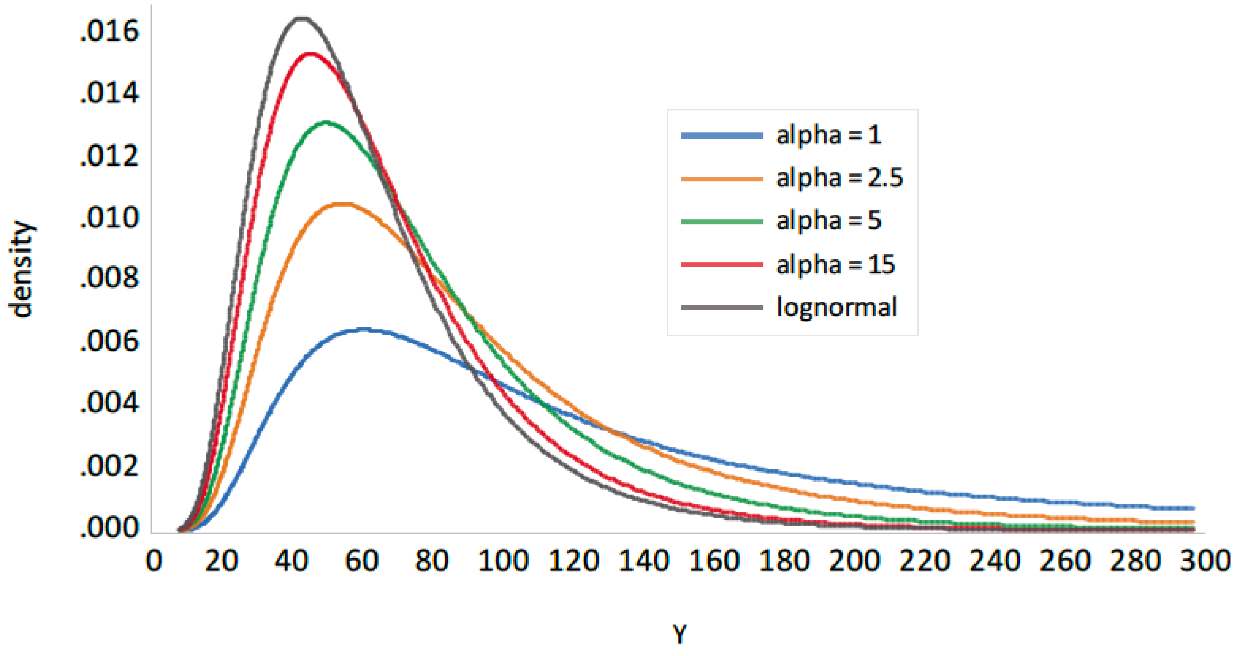


Figure 4. Probability density functions for a lognormal distribution with $\mu = 4$ and $\sigma = 0.5$, and Pareto-lognormal distributions with $\alpha = 1, 2.5, 5, 15$, $\mu = 4$ and $\sigma = 0.5$.

Pareto-Lognormal distribution

Note that as α becomes large, PLN tends towards lognormal distribution. Smaller values of α lead to distributions with fat tails.

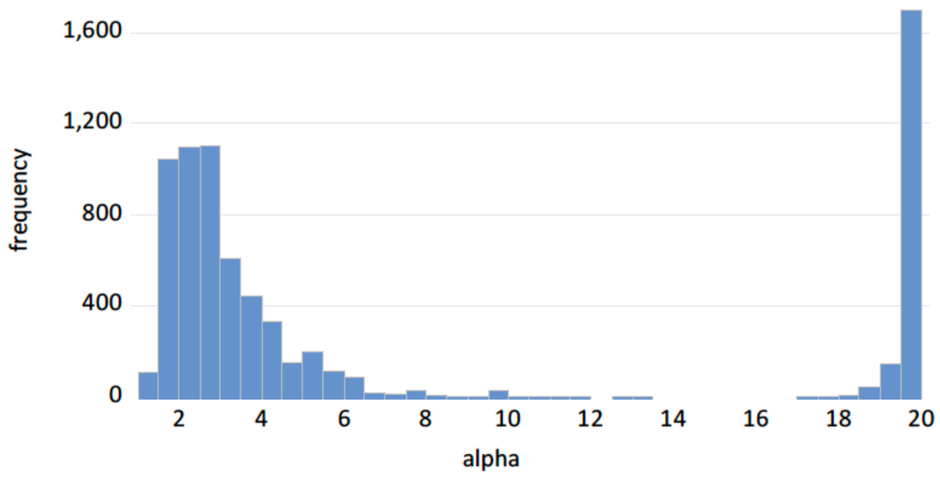


Figure 2. Histogram of UQICD estimates of α for all country/year Pareto-lognormal distributions.

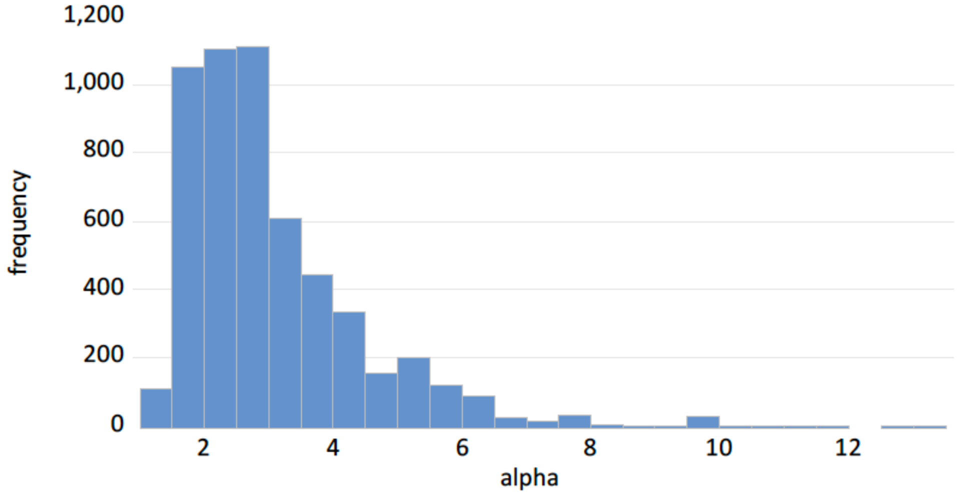


Figure 3. Histogram of Pareto-lognormal UQICD estimates of α that are less than 15.

Pareto-lognormal distribution

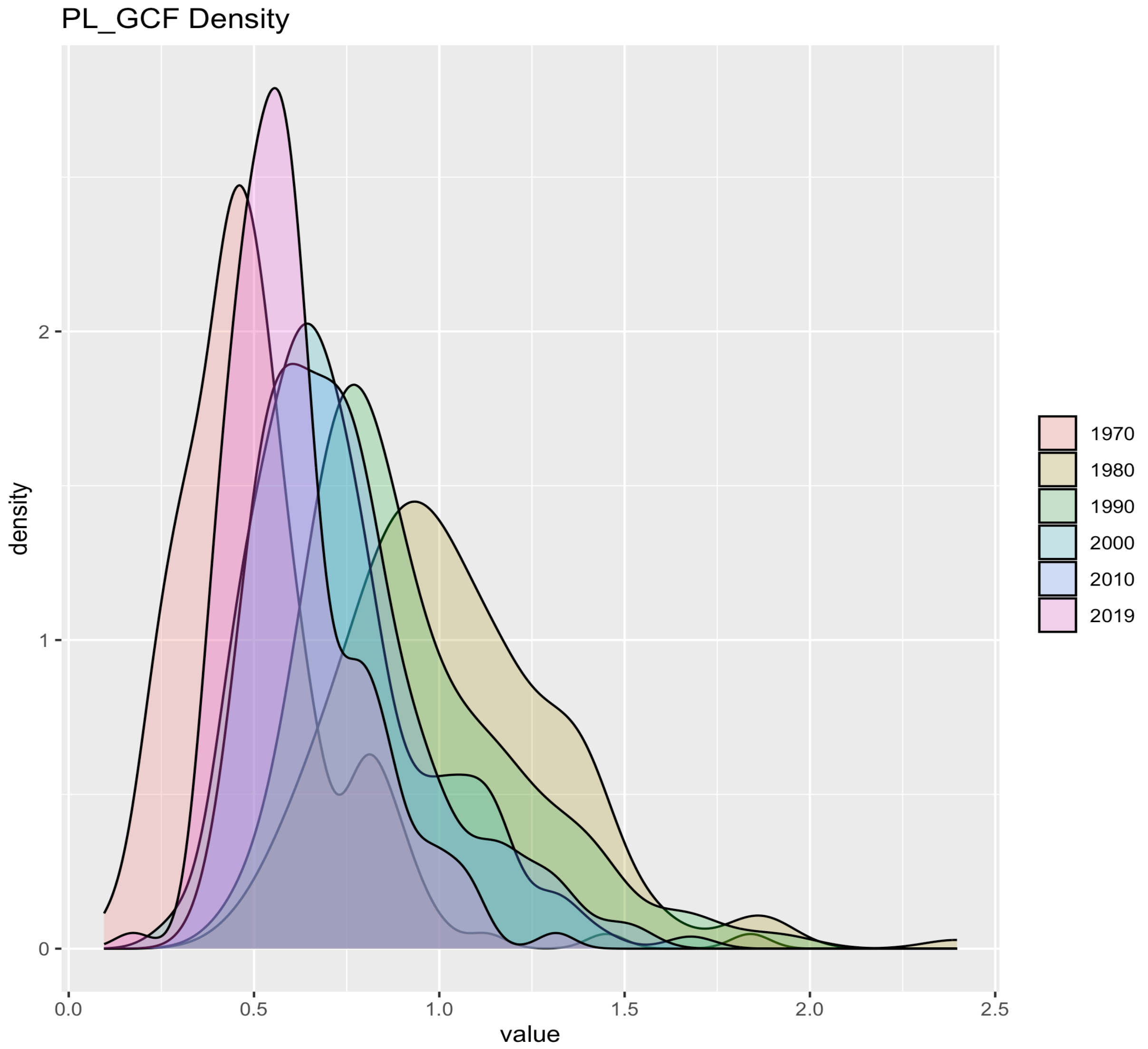
We found:

- **74% of all estimates of α are less than or equal to 15.**
- **Minimum value of α is 1.3; mean 3.0 and median 2.6.**
- **These are distributions with fat tails**
- **Need values of α > 1 for mean to exist; and α > 2 for variance to exist.**
- **All the estimates of α guarantee the existence of mean but 16% distributions do not have variance.**

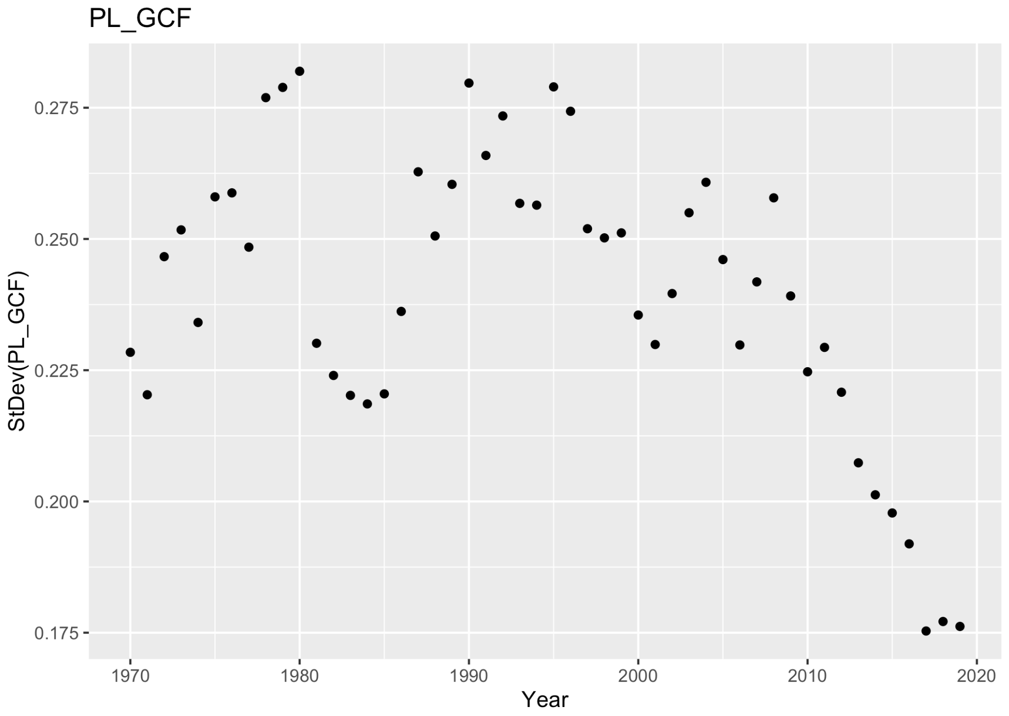
Directions for further work

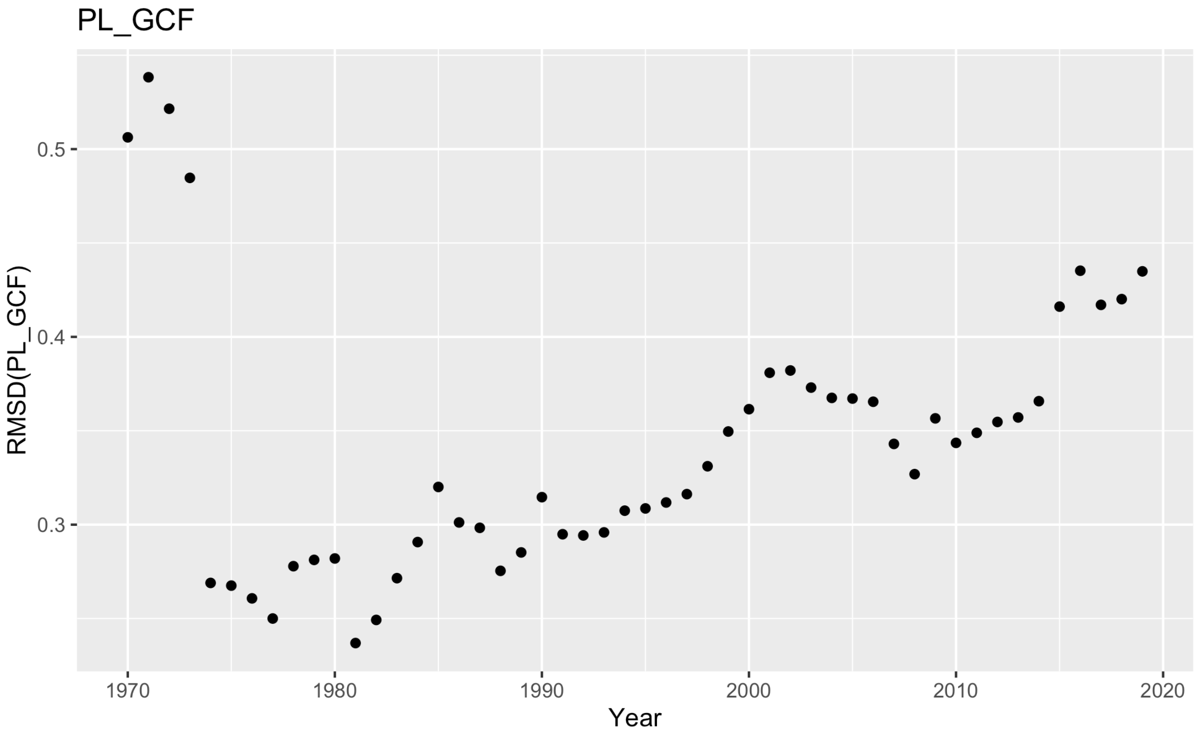
- Convergence in income distributions
  - Use Kullback-Leibler distance between distributions
  - Create a summary measure from the matrix of disturbances.
- Growth incidence curves
  - Examine growth incidence curves over different time periods
  - Pro-poor growth
- Convergence in real incomes
  - Revisit Barro (1991)
- Convergence in price levels
  - Analysis of price levels

Price levels - GCF

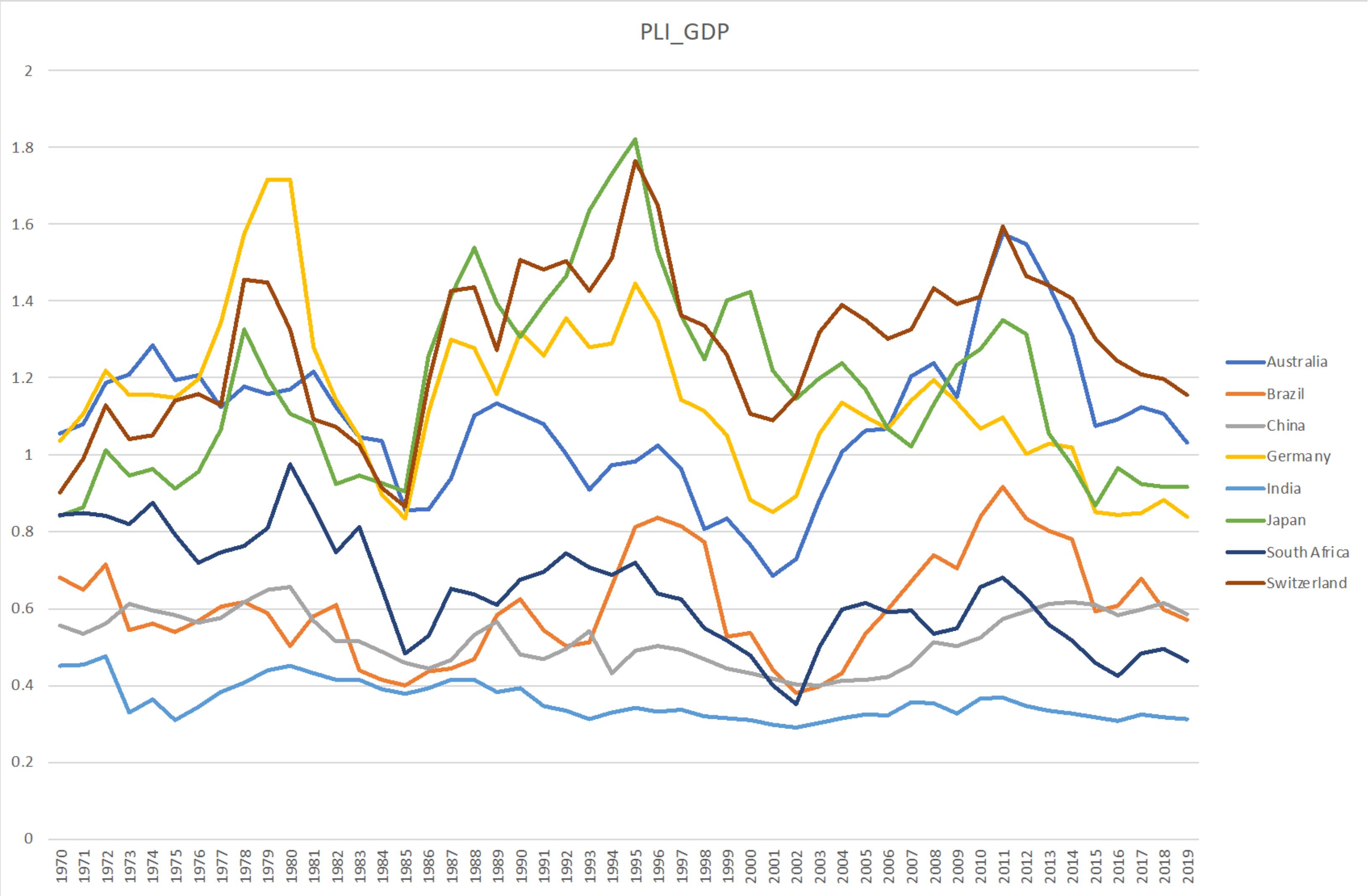


Price levels

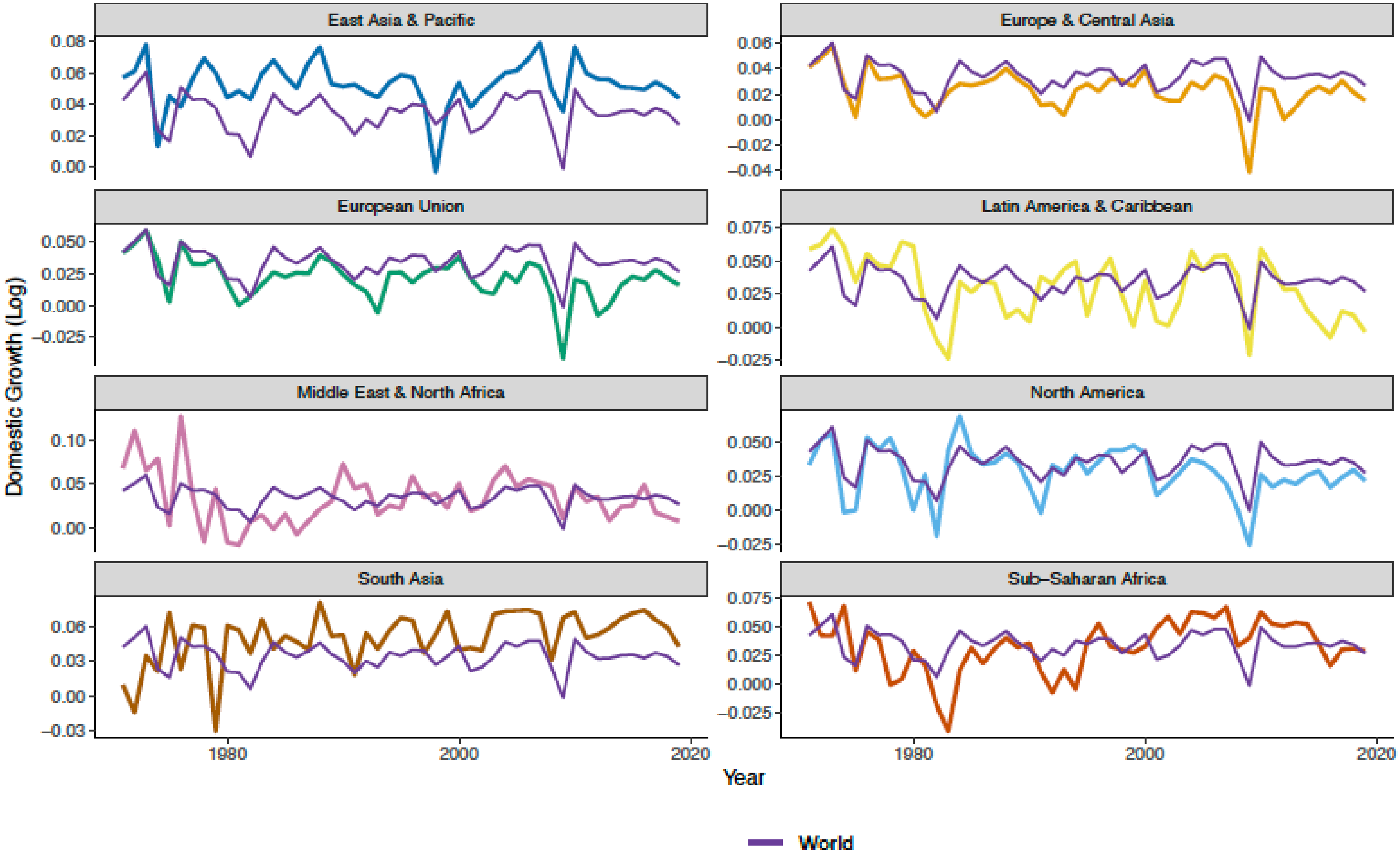




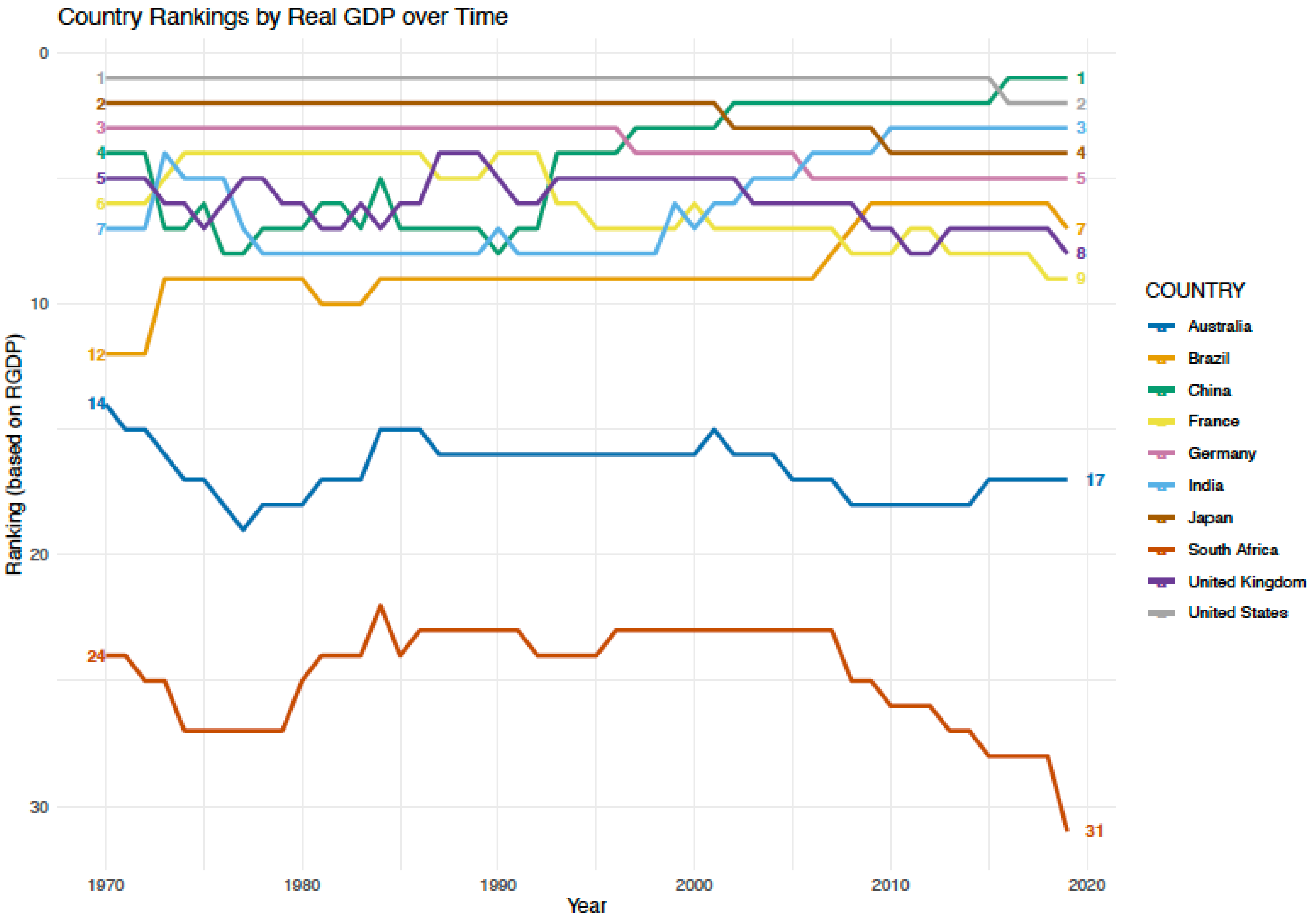
Price level indices for GDP - selected countries



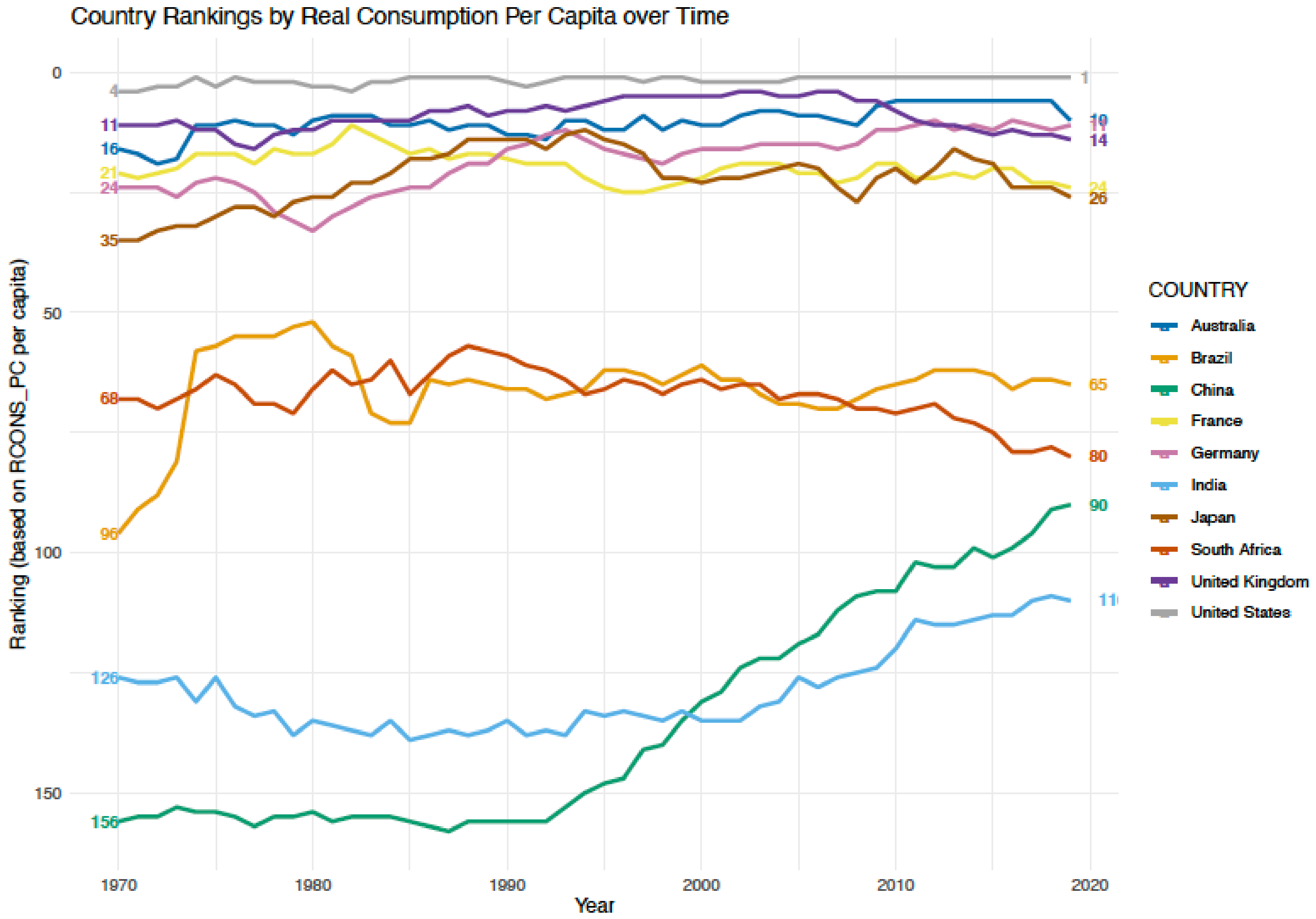
Growth: Geographical regions



Rankings of countries



Rankings of countries



Rankings of countries